

General Operator Solutions and BRST Quantization of Superstrings in the pp -Wave with Torsion

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Abstract

We completely accomplish the canonically covariant quantization of Ramond-Neveu-Schwarz (RNS) superstrings in the pp -wave background with a non-zero flux of the NS-NS antisymmetric two-form field. Here this flux is equivalent to a nonvanishing torsion. In this quantization, general operator solutions, which satisfy the entire equation of motion and all the canonical (anti)commutation relations, play an important role. The whole of covariant string coordinates and fermions can be composed of free modes. Moreover, employing covariant free-mode representations, we calculate the anomaly in the super-Virasoro algebra and determine the number of dimensions of spacetime and the ordering constant from the nilpotency condition of the BRST charge in the pp -wave background with the flux.

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§1. Introduction

The perfect quantization of superstrings in a variety of backgrounds, for example curved spacetime, is of great importance. In particular, the canonical quantization is significant for searching the fundamental laws of physics, including the string landscape, the string field theory, the AdS/CFT correspondence and matrix models. Of course, quantization in a number of interesting backgrounds has already been investigated and has been applied to many models. However, the canonically covariant quantization, which is the point to understand the fundamental structure, of superstrings in the curved spacetime background is not yet accomplished. Therefore, it is necessary to do new development, and it is effective to make a thorough investigation of the canonically covariant quantization for superstrings in the simple curved spacetime background.

The purpose of this paper is to canonically quantize the Ramond-Neveu-Schwarz (RNS) superstring in the pp -wave background with a non-zero flux of the NS-NS antisymmetric two-form field, by using the covariant BRST operator formalism. The method of the covariant BRST quantization of bosonic strings in the pp -wave background with the flux, which we have already performed,¹⁾ is very useful for canonical quantization of the superstrings. Moreover, it is particularly important to construct general operator solutions and covariant free-mode representations. First, we construct the general operator solutions which satisfy the entire equation of motion and all the canonical (anti)commutation relations. Here the equations of motion mean the Heisenberg equations of motion whose form is that of the Euler-Lagrange equations of motion in the pp -wave background with the flux. The general operator solutions of covariant string coordinates and fermions can be composed in the covariant free-mode representations. Second, by using the free-mode representations of the covariant string coordinates and fermions for the energy-momentum tensor and the supercurrent, we calculate the anomaly in the super-Virasoro algebra and determine the number of dimensions of spacetime and the ordering constant from the nilpotency condition of the BRST charge in the pp -wave background with the flux. Our covariant quantization has a deep meaning and will bring a new effect, though there are other methods,^{2)–13)} for example, the light-cone quantization and the conformal field theory.

This paper is organized as follows. In §2 we briefly review the action of the RNS superstring in general background fields and the pp -wave background with the flux. In §3 we obtain the equations of motion of a closed superstring in the pp -wave background with the flux and construct the general solutions of them. In §4 we make Dirac brackets in the general background by using the second-class constraints for Majorana fermions, and we complete all the canonical (anti)commutation relations.

We also explain the canonical anticommutation relations of the Ramond (R) fermions and the Neveu-Schwarz (NS) fermions. It is important to note the anti-periodic delta function appearing in the NS sector. In §5 we present new free-mode representations of all the covariant string coordinates and fermions, and we write up the definition of the normal orderings about the modes of twisted fields. The general operator solutions are explained in §4 and §5. In §6 we prove that both the general operator solutions and the free-mode representations satisfy all the equal-time canonical (anti)commutation relations among all the covariant string coordinates and fermions. In §7 we calculate the anomaly in the super-Virasoro algebra by using the energy-momentum tensor and the supercurrent in the free-mode representations of all the covariant string coordinates and fermions. We also comment on the super-Virasoro algebra of ghosts and antighosts. In §8 we determine the number of dimensions of spacetime and the ordering constant from the nilpotency condition of the BRST charge in the pp -wave background with the flux. Section 9 contains some conclusions.

§2. Action and Backgrounds

Before discussing our models, we explain our notation. We take as the flat world-sheet metric $\eta^{ab} = \text{diag}(-1, +1)$ and choose the representation of the two dimensional Dirac matrices $\gamma^0 = i\sigma^2$, $\gamma^1 = \sigma^1$. Moreover we define $\gamma_3 = \gamma^0\gamma^1 = \sigma^3$. Here σ^i ($i = 1, 2, 3$) are Pauli matrices. The Dirac matrices obey the Dirac algebra $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$ and satisfy the following relation $\gamma^a\gamma^b = \eta^{ab} + \epsilon^{ab}\gamma_3$, where ϵ^{ab} is the totally world-sheet anti-symmetric tensor ($\epsilon^{01} = +1$). We define a two dimensional Majorana spinor $\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$, and we use the components of the Majorana spinor as ψ_A ($A = +, -$). The Dirac conjugate of ψ is $\bar{\psi} = \psi^\dagger\gamma^0 = \psi^T\gamma^0 = (-\psi_-, \psi_+)$.

We consider the RNS superstring in the background fields which are a general curved spacetime metric $G_{\mu\nu}$ and a NS-NS anti-symmetric tensor field $B_{\mu\nu}$. Our starting point is the total action $S = S_M + S_{\text{GF+gh}}$, where S_M has the local supersymmetry and the total action S is BRST invariant. The action S_M with the string coordinates, the Majorana fermions, the zweibein and the world-sheet gravitino is^{14)–19)}

$$\begin{aligned}
S_M = & -\frac{1}{4\pi\alpha'} \int d\tau d\sigma e \left[(g^{\alpha\beta} G_{\mu\nu} + e^{-1} \epsilon^{\alpha\beta} B_{\mu\nu}) \partial_\alpha X^\mu \partial_\beta X^\nu + i G_{\mu\nu} \bar{\psi}^\mu \gamma^\alpha \mathcal{D}_\alpha \psi^\nu \right. \\
& + \frac{1}{8} \mathcal{R}_{\mu\rho\nu\sigma} \bar{\psi}^\mu (1 + \gamma_3) \psi^\nu \bar{\psi}^\rho (1 + \gamma_3) \psi^\sigma \\
& \left. + 2i G_{\mu\nu} \bar{\chi}_\alpha \gamma^\beta \gamma^\alpha \psi^\mu \partial_\beta X^\nu + \frac{1}{2} G_{\mu\nu} \bar{\chi}_\alpha \gamma^\beta \gamma^\alpha \chi_\beta \bar{\psi}^\mu \psi^\nu + \frac{1}{6} H_{\mu\nu\rho} \bar{\chi}_\alpha \gamma^\beta \gamma^\alpha \psi^\mu \bar{\psi}^\nu \gamma_\beta \gamma_3 \psi^\rho \right]. \quad (2.1)
\end{aligned}$$

The action $S_{\text{GF+gh}}$ with ghosts and antighosts is

$$S_{\text{GF+gh}} = \frac{1}{2\pi} \int d\tau d\sigma e \left[\mathcal{B}_{1\alpha}^a (e_a^\alpha - \delta_a^\alpha) + \bar{\mathcal{B}}_2^\alpha \chi_\alpha - i b_{\alpha\beta} \nabla^\alpha c^\beta + \bar{\beta}_\alpha \nabla^\alpha \gamma \right], \quad (2.2)$$

where the action includes the gauge-fixing condition and the antighosts are restricted from the conditions of the Weyl symmetry, the super-Weyl symmetry and the local Lorentz symmetry. We have to note that the indices a, b denote the local Lorentz world-sheet indices which run over 0,1, the greek indices α, β denote the curved world sheet indices which run over 0,1 and the greek indices $\mu, \nu, \rho, \sigma, \lambda, \omega$ denote the spacetime indices which run over $0, 1, \dots, D-1$. The fields which appear on this world-sheet are string coordinates (world-sheet bosonic fields) X^μ and their superpartners (world-sheet Majorana fermionic fields) ψ_A^μ , zweibein e_α^a and their fermionic superpartners (gravitino) χ_α . We define the curved world-sheet metric $g_{\alpha\beta}$ and the determinant of the zweibein as follows: $g_{\alpha\beta} = e_\alpha^a e_\beta^b \eta_{ab}$, $e = \det e_\alpha^a$. Moreover we had better define the inverse of the zweibein as $e_a^\alpha = (e_\alpha^a)^{-1}$ for useful. Then Dirac matrices in curved world-sheets become $\gamma^\alpha = e_a^\alpha \gamma^a$. Here we define the covariant derivative \mathcal{D}_α and the generalized Riemann tensor $\mathcal{R}_{\mu\rho\nu\sigma}$, which contain the flux of the NS-NS anti-symmetric field, as follows:

$$\mathcal{D}_\alpha \psi^\nu = \partial_\alpha \psi^\nu + \left(\Gamma_{\rho\sigma}^\nu + \frac{1}{2} \gamma_3 H_{\rho\sigma}^\nu \right) \partial_\alpha X^\rho \psi^\sigma, \quad (2.3)$$

$$\mathcal{R}_{\mu\rho\nu\sigma} = R_{\mu\rho\nu\sigma} + \frac{1}{2} (\nabla_\nu H_{\mu\rho\sigma} - \nabla_\sigma H_{\mu\rho\nu}) + \frac{1}{4} G_{\lambda\omega} (H_{\rho\nu}^\lambda H_{\mu\sigma}^\omega - H_{\rho\sigma}^\lambda H_{\mu\nu}^\omega) \quad (2.4)$$

We use the Christoffel symbol $\Gamma_{\rho\sigma}^\mu$, the field strength of the NS-NS anti-symmetric field $H_{\mu\nu\rho}$, the ordinary Riemann tensor $R_{\mu\nu\rho\sigma}$ and the spacetime covariant derivative of the NS-NS flux $\nabla_\mu H_{\nu\rho\sigma}$, whose definitions are as follows:

$$\Gamma_{\rho\sigma}^\mu = \frac{1}{2} G^{\mu\nu} (\partial_\sigma G_{\nu\rho} + \partial_\rho G_{\nu\sigma} - \partial_\nu G_{\rho\sigma}), \quad (2.5)$$

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}, \quad (2.6)$$

$$R_{\mu\nu\rho\sigma} = G_{\mu\lambda} (\partial_\rho \Gamma_{\nu\sigma}^\lambda - \partial_\sigma \Gamma_{\nu\rho}^\lambda + \Gamma_{\omega\rho}^\lambda \Gamma_{\nu\sigma}^\omega - \Gamma_{\omega\sigma}^\lambda \Gamma_{\nu\rho}^\omega), \quad (2.7)$$

$$\nabla_\mu H_{\nu\rho\sigma} = \partial_\mu H_{\nu\rho\sigma} - \Gamma_{\nu\mu}^\lambda H_{\lambda\rho\sigma} - \Gamma_{\rho\mu}^\lambda H_{\nu\lambda\sigma} - \Gamma_{\sigma\mu}^\lambda H_{\nu\rho\lambda}. \quad (2.8)$$

In the action $S_{\text{GF+gh}} = S_{\text{GF}} + S_{\text{gh}}$, S_{GF} is the gauge fixing action and S_{gh} is the Fadeev-Popov ghost action. $\mathcal{B}_{1\alpha}^a$ and \mathcal{B}_2^α are the auxiliary fields to fix the gauge. c^α , $b_{\alpha\beta}$ and γ , β_α are, respectively, the ghost field, the antighost field and their superpartners. Since we achieve the covariant BRST quantization for string theory in this paper, we choose the covariant gauge-fixing condition on the world-sheet; $e_a^\alpha = \delta_a^\alpha$ and $\chi_\alpha = 0$. These covariant gauge-fixing conditions are given by the equations of motion for the auxiliary fields $\mathcal{B}_{1\alpha}^a$ and \mathcal{B}_2^α . Using these gauge-fixing conditions, the

zweibein field and the gravitino field vanish from the action, so that the action has only X^μ and ψ_A^μ . After gauge fixing, we use the world-sheet light-cone coordinates $\sigma^\pm = \tau \pm \sigma$, so that the components of the world-sheet metric and the world-sheet totally antisymmetric tensor become $\eta^{+-} = \eta^{-+} = -2$ and $\epsilon^{+-} = -\epsilon^{-+} = -2$. Also, their partial derivatives are then $\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$. Moreover, we use the spacetime light-cone coordinates $X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^1)$ and the spacetime light-cone components of Majorana fermion $\psi_A^\pm = \frac{1}{\sqrt{2}}(\psi_A^0 \pm \psi_A^1)$.

The condition that superstring theory be Weyl-invariant in quantization on the world-sheet requires that the renormalization group β functions must vanish at all loop orders; these necessary conditions correspond to the field equations, which resembles Einstein's equation, the antisymmetric tensor generalization of Maxwell's equation and so on.²⁰⁾ As the background fields which satisfy these field equations, we use the following pp -wave metric and antisymmetric tensor field, whose flux is a constant:

$$ds^2 = -\mu^2(X^2 + Y^2)dX^+dX^+ - 2dX^+dX^- + dXdX + dYdY + dX^kdX^k, \quad (2.9)$$

$$B = -\mu Y dX^+ \wedge dX + \mu X dX^+ \wedge dY, \quad (2.10)$$

where we define $X^{\mu=2} = X$ and $X^{\mu=3} = Y$, and the index k runs over $4, 5, \dots, D-1$, that is to say, the components of $G_{\mu\nu}$ and $B_{\mu\nu}$ are

$$G_{++} = -\mu^2(X^2 + Y^2), \quad G_{+-} = G_{-+} = -1, \quad (2.11)$$

$$G_{ij} = \delta_{ij}, \quad i, j = 2, 3, \dots, D-1, \quad (2.12)$$

$$B_{+2} = -B_{2+} = -\mu Y, \quad B_{+3} = -B_{3+} = \mu X, \quad (2.13)$$

with all others vanishing. In this NS-NS pp -wave background, the generalized Riemann tensor becomes $\mathcal{R}_{\mu\rho\nu\sigma} = 0$ because of the existence of the NS-NS flux, however this spacetime is highly curved at the standpoint of the ordinary Riemann tensor $R_{\mu\rho\nu\sigma}$. Finally, we introduce the complex coordinates $Z = X + iY$, $Z^* = X - iY$ ^{*)}, and the spacetime complex components of Majorana fermion $\psi_A^Z = \psi_A^2 + i\psi_A^3$, $\psi_A^{Z^*} = \psi_A^2 - i\psi_A^3$. Using the complex coordinates the pp -wave metric become $G_{++} = -\mu^2 Z^* Z$, $G_{ZZ^*} = G_{Z^*Z} = \frac{1}{2}$ and the NS-NS field become $B_{+Z} = -B_{Z+} = -\frac{i}{2}\mu Z^*$, $B_{+Z^*} = -B_{Z^*+} = \frac{i}{2}\mu Z$. Moreover we introduce the world-sheet covariant derivatives $D_\pm = \partial_\pm \pm i\mu\partial_\pm X^+$. Then the action S_M becomes the following form:

$$S_M = \frac{1}{2\pi\alpha'} \int d\tau d\sigma \left[-2\partial_+ X^+ \partial_- X^- - 2\partial_- X^+ \partial_+ X^- + (D_+ Z)^* D_- Z + (D_- Z)^* D_+ Z + 2\partial_+ X^k \partial_- X^k \right]$$

^{*)} In our previous paper,¹⁾ we used the notation of \bar{Z} for the complex conjugate of Z , however in this paper we use the notation of bar for the Dirac conjugate of the fermionic field. Therefore we use the notation of Z^* for the complex conjugate of Z in this paper.

$$\begin{aligned}
& -i\psi_+^+\partial_-\psi_+^- - i\psi_-^+\partial_+\psi_-^- - i\psi_+^-\partial_-\psi_+^+ - i\psi_-^-\partial_+\psi_-^+ \\
& + \frac{i}{2} \{ \psi_+^{Z*} D_- \psi_+^Z + \psi_+^Z (D_- \psi_+^Z)^* + \psi_-^{Z*} D_+ \psi_-^Z + \psi_-^Z (D_+ \psi_-^Z)^* \} \\
& + \mu \psi_+^+ \{ \psi_+^{Z*} D_- Z - \psi_+^Z (D_- Z)^* \} - \mu \psi_-^+ \{ \psi_-^{Z*} D_+ Z - \psi_-^Z (D_+ Z)^* \} \\
& - i\mu^2 Z^* Z (\psi_+^+ \partial_- \psi_+^+ + \psi_-^+ \partial_+ \psi_-^+) + i\psi_+^k \partial_- \psi_+^k + i\psi_-^k \partial_+ \psi_-^k \Big], \tag{2.14}
\end{aligned}$$

where we remove $(\psi_\pm^\pm)^2$ from the action S_M because of $(\psi_\pm^\pm)^2 = 0$ in both quantum theory and classical theory. Here we note that the upper indices of ψ_A^μ always denote spacetime indices and lower indices of ψ_A^μ always denote world-sheet spinor indices. Do not confuse upper indices with lower indices.

§3. The equations of motion of X^μ and ψ_A^μ and their general solutions

We obtain the equations of motion of X^μ and ψ_A^μ from the action (2.14) ^{**)}. These equations are obviously related to the Heisenberg equations of motion with respect to quantization. The equations of X^μ and ψ_A^μ intricately interact with each other in the simple variation of the action. However we can remove fermionic fields ψ_A^μ from the equations of X^μ , using the equations of ψ_A^μ . Therefore the equations of X^μ become the same as the equations in our previous paper.¹⁾ It is no exaggeration to say that this enables us easily to quantize the case of the superstring in the NS-NS pp -wave background. Then we obtain the final equations of motion as follows.

- The equations of motion of X^+ and X^k are

$$\partial_+ \partial_- X^+ = 0, \quad \partial_+ \partial_- X^k = 0. \tag{3.1}$$

- The equations of motion of Z and Z^* are

$$D_+ D_- Z = 0, \quad D_+^* D_-^* Z^* = 0. \tag{3.2}$$

- The equation of motion of X^- is

$$\partial_+ \partial_- X^- + \frac{i\mu}{4} [\partial_+ (Z^* D_- Z - Z D_-^* Z^*) - \partial_- (Z^* D_+ Z - Z D_+^* Z^*)] = 0. \tag{3.3}$$

- The equations of motion of ψ_\pm^\pm and ψ_\pm^k are

$$\partial_- \psi_+^+ = 0, \quad \partial_+ \psi_-^+ = 0, \quad \partial_- \psi_+^k = 0, \quad \partial_+ \psi_-^k = 0. \tag{3.4}$$

^{**)} When we differentiate with the Grassmann number θ , we always differentiate from the right-hand side:

$$\frac{\partial(AB)}{\partial\theta} = A \frac{\partial B}{\partial\theta} + (-1)^{|B|} \frac{\partial A}{\partial\theta} B$$

where $|B|$ is the statistical factor of B

- The equations of motion of ψ_{\pm}^Z and ψ_{\pm}^{Z*} are

$$D_-(\psi_+^Z + i\mu\psi_+^+Z) = 0, \quad D_+(\psi_-^Z - i\mu\psi_-^+Z) = 0, \quad (3.5)$$

$$D_-^*(\psi_+^{Z*} - i\mu\psi_+^+Z^*) = 0, \quad D_+^*(\psi_-^{Z*} + i\mu\psi_-^+Z^*) = 0. \quad (3.6)$$

- The equations of motion of ψ_{\pm}^- are

$$\partial_- \psi_+^- + \frac{i}{2}\mu [\psi_+^{Z*} D_- Z - \psi_+^Z (D_- Z)^*] + \frac{1}{2}\mu^2 \partial_- (Z^* Z) \psi_+^+ = 0, \quad (3.7)$$

$$\partial_+ \psi_-^- - \frac{i}{2}\mu [\psi_-^{Z*} D_+ Z - \psi_-^Z (D_+ Z)^*] + \frac{1}{2}\mu^2 \partial_+ (Z^* Z) \psi_-^+ = 0. \quad (3.8)$$

Firstly we can solve the equations of bosonic fields, using the same method of the previous paper.¹⁾ The general solutions of bosonic fields are as follows:

$$X^+(\tau, \sigma) = X_L^+(\sigma^+) + X_R^+(\sigma^-), \quad (3.9)$$

$$X^k(\tau, \sigma) = X_L^k(\sigma^+) + X_R^k(\sigma^-), \quad (3.10)$$

$$Z(\tau, \sigma) = e^{-i\mu\tilde{X}^+} [f(\sigma^+) + g(\sigma^-)], \quad (3.11)$$

$$Z^*(\tau, \sigma) = e^{i\mu\tilde{X}^+} [f^*(\sigma^+) + g^*(\sigma^-)], \quad (3.12)$$

$$X^-(\tau, \sigma) = X_L^-(\sigma^+) + X_R^-(\sigma^-) + \frac{i}{2} [f(\sigma^+)g^*(\sigma^-) - f^*(\sigma^+)g(\sigma^-)], \quad (3.13)$$

where L and R indicate the left-moving and right-moving parts, respectively. Here we define

$$\tilde{X}^+ = X_L^+ - X_R^+. \quad (3.14)$$

We note that \tilde{X}^+ is not the periodic function, so that the arbitrary functions $f(\sigma^+)$ and $g(\sigma^-)$ satisfy the twisted boundary conditions.

Secondly we solve the equations of fermionic fields. The easiest equations are ψ_{\pm}^+ and ψ_{\pm}^k . Thus the solutions are

$$\psi_+^+(\tau, \sigma) = \psi_+^+(\sigma^+), \quad \psi_-^+(\tau, \sigma) = \psi_-^+(\sigma^-), \quad (3.15)$$

$$\psi_+^k(\tau, \sigma) = \psi_+^k(\sigma^+), \quad \psi_-^k(\tau, \sigma) = \psi_-^k(\sigma^-). \quad (3.16)$$

Needless to say, these fields are free fields. In the next place we solve the equations of ψ_{\pm}^Z and ψ_{\pm}^{Z*} . Because we can obtain the solutions of ψ_{\pm}^{Z*} from Hermitian conjugate of ψ_{\pm}^Z , it is enough to solve the equations of ψ_{\pm}^Z . Here we define $\Psi_{\pm}^Z = \psi_{\pm}^Z \pm i\mu\psi_{\pm}^+Z$. The differential equations of (3.5) become easier:

$$(\partial_{\mp} \mp i\mu\partial_{\mp} X^+) \Psi_{\pm}^Z = 0. \quad (3.17)$$

The general solutions are

$$\Psi_{\pm}^Z = e^{-i\mu\tilde{X}^+} \lambda_{\pm}(\sigma^{\pm}), \quad (3.18)$$

where λ_+ and λ_- are arbitrary Grassmann function of σ^+ and arbitrary Grassmann function of σ^- respectively. Here we may find the solutions which have only $e^{-i\mu X_L^+}$ or $e^{-i\mu X_R^+}$; however, these solutions are difficult to quantize, and we had better choose the solutions which have the same the factor as Z from the standpoint of supersymmetry. Therefore we obtain the general solutions of ψ_{\pm}^Z and ψ_{\pm}^{Z*} :

$$\psi_{\pm}^Z(\tau, \sigma) = e^{-i\mu\tilde{X}^+} \lambda_{\pm}(\sigma^{\pm}) \mp i\mu\psi_{\pm}^+(\sigma^{\pm})Z(\tau, \sigma), \quad (3.19)$$

$$\psi_{\pm}^{Z*}(\tau, \sigma) = e^{i\mu\tilde{X}^+} \lambda_{\pm}^*(\sigma^{\pm}) \pm i\mu\psi_{\pm}^+(\sigma^{\pm})Z^*(\tau, \sigma), \quad (3.20)$$

where ψ_{\pm}^{Z*} is obtained by taking the Hermitian conjugate of ψ_{\pm}^Z . Moreover substituting the general solution of Z and Z^* , we can represent ψ_{\pm}^Z and ψ_{\pm}^{Z*} by using the twisted fields f and g :

$$\psi_+^Z(\tau, \sigma) = e^{-i\mu\tilde{X}^+} [\lambda_+(\sigma^+) - i\mu\psi_+^+(\sigma^+) \{f(\sigma^+) + g(\sigma^-)\}], \quad (3.21)$$

$$\psi_-^Z(\tau, \sigma) = e^{-i\mu\tilde{X}^+} [\lambda_-(\sigma^-) + i\mu\psi_-^+(\sigma^-) \{f(\sigma^+) + g(\sigma^-)\}], \quad (3.22)$$

$$\psi_+^{Z*}(\tau, \sigma) = e^{i\mu\tilde{X}^+} [\lambda_+^*(\sigma^+) + i\mu\psi_+^+(\sigma^+) \{f^*(\sigma^+) + g^*(\sigma^-)\}], \quad (3.23)$$

$$\psi_-^{Z*}(\tau, \sigma) = e^{i\mu\tilde{X}^+} [\lambda_-^*(\sigma^-) - i\mu\psi_-^+(\sigma^-) \{f^*(\sigma^+) + g^*(\sigma^-)\}]. \quad (3.24)$$

We can confirm the existence of the supersymmetry between Z and ψ_{\pm}^Z from direct calculation.

Finally we solve the equations of ψ_{\pm}^- . As a matter of fact, using the equations of motion of ψ_{\pm}^+ , ψ_{\pm}^Z , the equations of motion of ψ_{\pm}^- become easier forms:

$$\partial_- \psi_+^- = \partial_- \left[-\frac{i}{2}(Z\psi_+^{Z*} - Z^*\psi_+^Z) - \mu^2\psi_+^+ Z^* Z \right], \quad (3.25)$$

$$\partial_+ \psi_-^- = \partial_+ \left[\frac{i}{2}(Z\psi_-^{Z*} - Z^*\psi_-^Z) - \mu^2\psi_-^+ Z^* Z \right]. \quad (3.26)$$

Therefore we can solve these equations, removing the differential operator from the left-hand side.

The general solutions of ψ_{\pm}^- are

$$\psi_+^- = \psi_{0+}^- - \frac{i}{2}(Z\psi_+^{Z*} - Z^*\psi_+^Z) - \mu^2\psi_+^+ Z^* Z, \quad (3.27)$$

$$\psi_-^- = \psi_{0-}^- + \frac{i}{2}(Z\psi_-^{Z*} - Z^*\psi_-^Z) - \mu^2\psi_-^+ Z^* Z, \quad (3.28)$$

where ψ_{0+}^- and ψ_{0-}^- are an arbitrary Grassmann function of σ^+ and an arbitrary Grassmann function of σ^- respectively. Moreover substituting the general solutions of Z , Z^* , ψ_{\pm}^Z , ψ_{\pm}^{Z*} , we can

represent ψ_{\pm}^- by using the twisted fields f , g , λ_{\pm} as follows:

$$\psi_+^-(\tau, \sigma) = \psi_{0+}^-(\sigma^+) - \frac{i}{2}\mu [f(\sigma^+) + g(\sigma^-)] \lambda_+^*(\sigma^+) + \frac{i}{2}\mu [f^*(\sigma^+) + g^*(\sigma^-)] \lambda_+(\sigma^+), \quad (3.29)$$

$$\psi_-^-(\tau, \sigma) = \psi_{0-}^-(\sigma^-) + \frac{i}{2}\mu [f(\sigma^+) + g(\sigma^-)] \lambda_-^*(\sigma^-) - \frac{i}{2}\mu [f^*(\sigma^+) + g^*(\sigma^-)] \lambda_-(\sigma^-). \quad (3.30)$$

We can solve all the equations of motion of the superstring in the NS-NS pp -wave background generally and we represent all the general solutions by using the useful fields in spite of existence of interactions. After this section, we quantize these general solutions. Before we quantize these solutions, we have to consider the method to quantize superstrings in the background fields. Superstrings in the background fields have some constraints. In the following section, we consider the constraints and we construct the formula of canonical (anti)commutation relations by using the Dirac brackets.

§4. Constraints from Majorana fermion and Dirac bracket

We consider the second-class constraints from RNS superstrings in the background fields $G_{\mu\nu}$ and $B_{\mu\nu}$. The canonical momenta associated with string coordinates X^μ and Majorana fermions ψ_A^μ are defined as

$$P_\mu = \frac{\partial S_M}{\partial(\partial_\tau X^\mu)} = \frac{1}{2\pi\alpha'} \left[G_{\mu\nu} \partial_\tau X^\nu - B_{\mu\nu} \partial_\sigma X^\nu + \frac{i}{2} G_{\rho\omega} \psi^{\dagger\rho} (\Gamma_{\mu\nu}^\omega + \frac{\gamma_3}{2} H_{\mu\nu}^\omega) \psi^\nu \right], \quad (4.1)$$

$$\pi_{\mu A} = \frac{\partial S_M}{\partial(\partial_\tau \psi_A^\mu)} = \frac{i}{4\pi\alpha'} G_{\mu\nu} \psi_A^\nu, \quad (4.2)$$

where $\partial_\tau = \frac{\partial}{\partial\tau}$ and $\partial_\sigma = \frac{\partial}{\partial\sigma}$. In the case of the fermions, we always differentiate the action with the Grassmann number from right-hand side. Although the momenta P_μ contain $\partial_\tau X^\mu$ certainly, the momenta $\pi_{\mu A}$ do not contain $\partial_\tau \psi_A^\mu$. Therefore, they give rise to the second-class constraints $\phi_{\mu A} = \pi_{\mu A} - \frac{i}{4\pi\alpha'} G_{\mu\nu} \psi_A^\nu \approx 0$. Here we must note that $G_{\mu\nu}$ is a function of string coordinates X^μ . For this reason, Poisson brackets between the canonical momenta P_μ and the constraints $\phi_{\mu A}$ are not zero, therefore we also must consider the Dirac brackets about P_μ . The Poisson brackets between X^μ and $\phi_{\mu A}$ vanish, so that the Dirac brackets which contain X^μ become the Poisson brackets. The other Dirac brackets become as follows: *)

$$\{\psi_A^\mu(\tau, \sigma), \psi_B^\nu(\tau, \sigma')\}_D = \begin{cases} -i \cdot 2\pi\alpha' G^{\mu\nu} \delta_{AB} \delta(\sigma - \sigma') & : \text{R sector}, \\ -i \cdot 2\pi\alpha' G^{\mu\nu} \delta_{AB} \tilde{\delta}(\sigma - \sigma') & : \text{NS sector}, \end{cases} \quad (4.3)$$

$$\{\psi_A^\mu(\tau, \sigma), \pi_{\nu B}(\tau, \sigma')\}_D = \begin{cases} \frac{1}{2} \delta_\nu^\mu \delta_{AB} \delta(\sigma - \sigma') & : \text{R sector}, \\ \frac{1}{2} \delta_\nu^\mu \delta_{AB} \tilde{\delta}(\sigma - \sigma') & : \text{NS sector}, \end{cases} \quad (4.4)$$

*) In this paper, we leave out the description of τ in the right-hand side about the Dirac brackets and the canonical (anti)commutation relations.

$$\{\pi_{\mu A}(\tau, \sigma), \pi_{\nu B}(\tau, \sigma')\}_D = \begin{cases} \frac{i}{8\pi\alpha'} G_{\mu\nu} \delta_{AB} \delta(\sigma - \sigma') & : \text{R sector}, \\ \frac{i}{8\pi\alpha'} G_{\mu\nu} \delta_{AB} \tilde{\delta}(\sigma - \sigma') & : \text{NS sector}, \end{cases} \quad (4.5)$$

$$[P_\mu(\tau, \sigma), P_\nu(\tau, \sigma')]_D = -\frac{i}{8\pi\alpha'} \partial_\mu G_{\rho\lambda} \partial_\nu G_{\sigma\omega} G^{\rho\sigma} \psi_A^\lambda(\sigma') \psi_A^\omega(\sigma') \delta(\sigma - \sigma'), \quad (4.6)$$

$$[P_\mu(\tau, \sigma), \psi_A^\nu(\tau, \sigma')]_D = \frac{1}{2} \partial_\mu G_{\rho\sigma} G^{\rho\nu} \psi_A^\sigma(\sigma') \delta(\sigma - \sigma'), \quad (4.7)$$

$$[P_\mu(\tau, \sigma), \pi_{\nu A}(\tau, \sigma')]_D = -\frac{i}{8\pi\alpha'} \partial_\mu G_{\nu\rho} \psi_A^\rho(\sigma') \delta(\sigma - \sigma'). \quad (4.8)$$

Here $\delta(\sigma)$ denotes the ordinary periodic delta-function and $\tilde{\delta}(\sigma)$ denotes the anti-periodic delta-function. We must consider the difference between the Ramond (R) fermions and the Neveu-Schwarz (NS) fermions. We must especially take notice of the NS fermions which have the anti-periodicity. In the case of the R fermions we can identify the world-sheet coordinates σ and σ' using the property of the periodic delta-function, however in the case of the NS fermions we cannot do it simply because of the anti-periodicity. The relation between the NS fermion and the delta-functions is

$$\psi^\mu(\sigma') \delta(\sigma - \sigma') = \psi^\mu(\sigma) \tilde{\delta}(\sigma - \sigma'). \quad (4.9)$$

We can prove this relation (4.9) by using the Fourier expansions. Substituting the condition $\pi_{\mu A} = \frac{i}{4\pi\alpha'} G_{\mu\nu} \psi_A^\nu$ to the Dirac brackets $\{\psi_A^\mu(\tau, \sigma), \pi_{\nu B}(\tau, \sigma')\}_D$ and $\{\pi_{\mu A}(\tau, \sigma), \pi_{\nu B}(\tau, \sigma')\}_D$, these correspond to the Dirac bracket $\{\psi_A^\mu(\tau, \sigma), \psi_B^\nu(\tau, \sigma')\}_D$. Moreover using the Poisson bracket between P_μ and $G_{\mu\nu}$ and the Leibniz rule, the Dirac bracket $[P_\mu(\tau, \sigma), \pi_{\nu A}(\tau, \sigma')]_D$ corresponds to the Dirac bracket $[P_\mu(\tau, \sigma), \psi_A^\nu(\tau, \sigma')]_D$. They mean that we have only to consider the Dirac brackets between X^μ , P_μ , ψ_A^μ . Therefore we do not have to consider the Dirac bracket about $\pi_{\mu A}$.

We quantize this constraint system, using the ordinary procedure which we replace i times Dirac brackets with the canonical (anti)commutation relations. Let us treat the case of the pp -wave background. The canonical (anti)commutation relations are as follows:

- The canonical commutation relations between the bosonic fields and their momenta are

$$[X^\mu(\tau, \sigma), P_\nu(\tau, \sigma')] = i\delta^\mu_\nu \delta(\sigma - \sigma'), \quad (4.10)$$

$$[X^\mu(\tau, \sigma), X^\nu(\tau, \sigma')] = 0, \quad [P_\mu(\tau, \sigma), P_\nu(\tau, \sigma')] = 0. \quad (4.11)$$

- The canonical anticommutation relations between the fermionic fields are

$$\{\psi_A^-(\tau, \sigma), \psi_B^-(\tau, \sigma')\} = \begin{cases} 2\pi\alpha' \mu^2 Z^* Z \delta_{AB} \delta(\sigma - \sigma') & : \text{R sector}, \\ 2\pi\alpha' \mu^2 Z^* Z \delta_{AB} \tilde{\delta}(\sigma - \sigma') & : \text{NS sector}, \end{cases} \quad (4.12)$$

$$\{\psi_A^+(\tau, \sigma), \psi_B^-(\tau, \sigma')\} = \begin{cases} -2\pi\alpha' \delta_{AB} \delta(\sigma - \sigma') & : \text{R sector}, \\ -2\pi\alpha' \delta_{AB} \tilde{\delta}(\sigma - \sigma') & : \text{NS sector}, \end{cases} \quad (4.13)$$

$$\{\psi_A^Z(\tau, \sigma), \psi_B^{Z*}(\tau, \sigma')\} = \begin{cases} 4\pi\alpha'\delta_{AB}\delta(\sigma - \sigma') & : \text{R sector}, \\ 4\pi\alpha'\delta_{AB}\tilde{\delta}(\sigma - \sigma') & : \text{NS sector}, \end{cases} \quad (4.14)$$

$$\{\psi_A^k(\tau, \sigma), \psi_B^l(\tau, \sigma')\} = \begin{cases} 2\pi\alpha'\delta^{kl}\delta_{AB}\delta(\sigma - \sigma') & : \text{R sector}, \\ 2\pi\alpha'\delta^{kl}\delta_{AB}\delta(\sigma - \sigma') & : \text{NS sector}. \end{cases} \quad (4.15)$$

- The canonical commutation relations between the bosonic fields and the fermionic fields are

$$[X^\mu(\tau, \sigma), \psi_A^\nu(\tau, \sigma')] = 0, \quad (4.16)$$

$$[P_Z(\tau, \sigma), \psi_A^-(\tau, \sigma')] = \frac{i}{2}\mu^2 Z^* \psi_A^+(\sigma')\delta(\sigma - \sigma'), \quad (4.17)$$

$$[P_{Z^*}(\tau, \sigma), \psi_A^-(\tau, \sigma')] = \frac{i}{2}\mu^2 Z \psi_A^+(\sigma')\delta(\sigma - \sigma'), \quad (4.18)$$

with all other commutations vanishing. In the case of the NS fermion, the world-sheet coordinate of fermion must be σ' because of the anti-periodicity. The characteristic point is that the (anti)commutation relations between ψ_A^- , P_Z and P_{Z^*} do not vanish. We note that fortunately the commutation relations between the momenta vanish because of the inverse of metric $G^{++} = 0$ in the pp -wave background. Therefore we can quantize the bosonic fields using the same method of the previous paper.¹⁾ Of course when $\mu = 0$, we obtain ordinary canonical (anti)commutation relations of superstrings in the flat spacetime.

§5. Free-mode representation

In this section we construct the free-mode representations in which the general operator solutions satisfy all the canonical (anti)commutation relations. The detailed proof of the free-mode representations in the canonically covariant quantization is given in the next section. The free-mode representations of bosonic fields without X^- are the same as the previous paper.¹⁾ Because X^- interact with ψ_A^Z and ψ_A^{Z*} , we have to newly construct the free-mode representation of X^- from the commutation relations between X^- and ψ_A^Z , ψ_A^{Z*} . The free-mode representations of bosonic fields X^+ , X^k , f , g are as follows:

$$X^+ = x^+ + \frac{\alpha'}{2} p^+(\sigma^+ + \sigma^-) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \left[\tilde{\alpha}_n^+ e^{-in\sigma^+} + \alpha_n^+ e^{-in\sigma^-} \right], \quad (5.1)$$

$$X^k = x^k + \frac{\alpha'}{2} p^k(\sigma^+ + \sigma^-) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \left[\tilde{\alpha}_n^k e^{-in\sigma^+} + \alpha_n^k e^{-in\sigma^-} \right], \quad (5.2)$$

$$f(\sigma^+) = \sqrt{\alpha'} \sum_{n \in \mathbb{Z}} \frac{A_n}{\sqrt{|n - \hat{\mu}|}} e^{-i(n - \hat{\mu})\sigma^+}, \quad (5.3)$$

$$g(\sigma^-) = \sqrt{\alpha'} \sum_{n \in \mathbb{Z}} \frac{B_n}{\sqrt{|n + \hat{\mu}|}} e^{-i(n + \hat{\mu})\sigma^-}. \quad (5.4)$$

where $n \neq 0$ which is put under Σ means $n \in \mathbb{Z}(n \neq 0)$. Here the dimensionless factor $\hat{\mu}$ is newly defined as

$$\hat{\mu} = \mu \alpha' p^+. \quad (5.5)$$

We must note that $\hat{\mu}$ is not a constant, which depends on the momentum operator p^+ , so that the commutation relation $[x^-, \hat{\mu}] = -i\mu\alpha'$. By Hermitian conjugate of f and g , we can obtain f^* and g^* , whose modes are defined as A_N^\dagger and B_N^\dagger . In the next place, we construct the free-mode representation of fermionic fields. First the free-mode representation of ψ_\pm^\pm and ψ_\pm^k are the same as ordinary free fields:

$$\psi_+^\pm(\sigma^+) = \sqrt{\alpha'} \sum_{r \in \mathbb{Z} + \varepsilon} \tilde{\psi}_r^\pm e^{-ir\sigma^+}, \quad \psi_-^\pm(\sigma^-) = \sqrt{\alpha'} \sum_{r \in \mathbb{Z} + \varepsilon} \psi_r^\pm e^{-ir\sigma^-}, \quad (5.6)$$

$$\psi_+^k(\sigma^+) = \sqrt{\alpha'} \sum_{r \in \mathbb{Z} + \varepsilon} \tilde{\psi}_r^k e^{-ir\sigma^+}, \quad \psi_-^k(\sigma^-) = \sqrt{\alpha'} \sum_{r \in \mathbb{Z} + \varepsilon} \psi_r^k e^{-ir\sigma^-} \quad (5.7)$$

where $\varepsilon = 0$ in the R sector and $\varepsilon = \frac{1}{2}$ in the NS sector. Second, we present the free-mode representations of ψ_\pm^Z and ψ_\pm^{Z*} , which must satisfy the Ramond or Neveu-Schwarz boundary condition of the closed superstring theory. Because the factor $e^{-i\mu\tilde{X}^+(\tau, \sigma)}$ in ψ_\pm^Z is transformed into $e^{-2\pi i\hat{\mu}} \cdot e^{-i\mu\tilde{X}^+(\tau, \sigma)}$ under the shift $\sigma \rightarrow \sigma + 2\pi$, λ_\pm in ψ_\pm^Z are twisted fields. In other words, these fields satisfy the twisted boundary condition:

$$\lambda_+(\sigma^+ + 2\pi) = \begin{cases} +e^{2\pi i\hat{\mu}} \lambda_+(\sigma^+) & : \text{R sector}, \\ -e^{2\pi i\hat{\mu}} \lambda_+(\sigma^+) & : \text{NS sector}, \end{cases} \quad (5.8)$$

$$\lambda_-(\sigma^- - 2\pi) = \begin{cases} +e^{2\pi i\hat{\mu}} \lambda_-(\sigma^-) & : \text{R sector}, \\ -e^{2\pi i\hat{\mu}} \lambda_-(\sigma^-) & : \text{NS sector}. \end{cases} \quad (5.9)$$

Under the boundary conditions (5.8), (5.9), the free-mode representations of λ_\pm are

$$\lambda_+(\sigma^+) = \sqrt{2\alpha'} \sum_{r \in \mathbb{Z} + \varepsilon} \tilde{\lambda}_r e^{-i(r-\hat{\mu})\sigma^+}, \quad (5.10)$$

$$\lambda_-(\sigma^-) = \sqrt{2\alpha'} \sum_{r \in \mathbb{Z} + \varepsilon} \lambda_r e^{-i(r+\hat{\mu})\sigma^-}. \quad (5.11)$$

Thus we can obtain the free-mode representations of λ_\pm^* by taking Hermitian conjugate of λ_\pm :

$$\lambda_+^*(\sigma^+) = \sqrt{2\alpha'} \sum_{r \in \mathbb{Z} + \varepsilon} \tilde{\lambda}_r^\dagger e^{i(r-\hat{\mu})\sigma^+}, \quad (5.12)$$

$$\lambda_-^*(\sigma^-) = \sqrt{2\alpha'} \sum_{r \in \mathbb{Z} + \varepsilon} \lambda_r^\dagger e^{i(r+\hat{\mu})\sigma^-}. \quad (5.13)$$

Third we present the free-mode representation of ψ_A^- . We have already known the free-mode representation of f , g , λ_\pm and their Hermitian conjugate in ψ_A^- . Imposing the canonical (anti)commutation

relations on ψ_A^- , $\psi_{0\pm}^-$ must be absolutely free field, so that $\psi_{0\pm}^-$ does not contain other fields. Therefore the free-mode representations of $\psi_{0\pm}^-$ are the same as ordinary free fields:

$$\psi_{0+}^- = \sqrt{\alpha'} \sum_{r \in \mathbb{Z} + \varepsilon} \tilde{\psi}_r^- e^{-ir\sigma^+}, \quad (5.14)$$

$$\psi_{0-}^- = \sqrt{\alpha'} \sum_{r \in \mathbb{Z} + \varepsilon} \psi_r^- e^{-ir\sigma^-}. \quad (5.15)$$

Finally we explain the construction of the free-mode representation of X^- in detail. The canonical commutation relation between X^- and ψ_{\pm}^Z must be zero, therefore the commutation relations between X^- and λ_{\pm} must be

$$[X^-(\tau, \sigma), \lambda_{\pm}(\sigma'^{\pm})] = i\mu[X^-(\tau, \sigma), \tilde{X}^+(\sigma')] \lambda_{\pm}(\sigma'^{\pm}). \quad (5.16)$$

We divide $X_L^- + X_R^-$ into an almost free part, X_0^- *) and a completely non-free part, X_1^- . Moreover we divide X_1^- into the bosonic part X_{1B}^- which is constructed from f and g and the fermionic part X_{1F}^- which is constructed from λ_{\pm} . The free-mode representation of X_{1B}^- is the same as that of our previous paper, and we can obtain X_{1F}^- using the same method of our previous paper.¹⁾ Although we did not explain in detail how to construct the free-mode representation of X^- in the previous paper, here, we explain how to construct it. The mode expansion of X_{1F}^- is

$$X_{1F}^- = \alpha' p_F^- \tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \left[\tilde{\alpha}_{Fn}^- e^{-in\sigma^+} + \alpha_{Fn}^- e^{-in\sigma^-} \right]. \quad (5.17)$$

Here we note that X_{1F}^- do not contain the zero mode x^- , which is only contained in X_0^- . The commutation relation $[X^-(\tau, \sigma), \tilde{X}^+(\tau, \sigma')]$ is

$$\begin{aligned} [X^-(\tau, \sigma), \tilde{X}^+(\tau, \sigma')] &= [X_0^-(\tau, \sigma), \tilde{X}^+(\tau, \sigma')] \\ &= -i\alpha'\sigma' + \alpha' \sum_{n \neq 0} \frac{1}{n} e^{in(\sigma - \sigma')} \end{aligned} \quad (5.18)$$

Calculating commutation relation (5.16) and comparing the left-hand side with the right-hand side, we can obtain the commutation relation between modes:

$$[p_F^-, \tilde{\lambda}_r] = -\mu \tilde{\lambda}_r, \quad [p_F^-, \lambda_r] = \mu \lambda_r, \quad (5.19)$$

$$[\tilde{\alpha}_{Fn}^-, \tilde{\lambda}_r] = -\mu \sqrt{2\alpha'} \tilde{\lambda}_{n+r}, \quad [\alpha_{Fn}^-, \lambda_r] = \mu \sqrt{2\alpha'} \lambda_{n+r}. \quad (5.20)$$

*) “almost free” means that only the zero-mode of X_0^- dose not commute with the twisted fields, namely f , g , and λ_{\pm} .

Therefore we can represent the modes p_F^- , $\tilde{\alpha}_{Fn}^-$, α_{Fn}^- by the modes $\tilde{\lambda}_r$ and λ_r , using the anticommutation relations between the modes λ_r , λ_r^\dagger (Eq.(5.38)) which are written up later:

$$p_F^- = \mu \sum_{r \in \mathbb{Z} + \varepsilon} (: \tilde{\lambda}_r^\dagger \tilde{\lambda}_r : - : \lambda_r^\dagger \lambda_r :), \quad (5.21)$$

$$\tilde{\alpha}_{Fn}^- = \mu \sqrt{2\alpha'} \sum_{r \in \mathbb{Z} + \varepsilon} \tilde{\lambda}_r^\dagger \tilde{\lambda}_{n+r}, \quad \alpha_{Fn}^- = -\mu \sqrt{2\alpha'} \sum_{r \in \mathbb{Z} + \varepsilon} \lambda_r^\dagger \lambda_{n+r}, \quad (5.22)$$

where the notation $::$ represents the normal ordered product, whose definition is given in the final part of this section (Eqs.(5.40)-(5.43)). Substituting these modes (5.21), (5.22) into (5.17), we can obtain the free-mode representation of X_{1F}^- . Thus we can obtain the free-mode representation of X^- using X_0^- , X_{1B}^- , X_{1F}^- and X_2^- , where $X_2^- = \frac{i\mu}{2}(fg^* - f^*g)$:

$$X^- = X_0^- + X_{1B}^- + X_{1F}^- + X_2^-. \quad (5.23)$$

The almost free part X_0^- is

$$X_0^- = x^- + \frac{1}{2}\alpha' p^- (\sigma^+ + \sigma^-) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} [\tilde{\alpha}_n^- e^{-in\sigma^+} + \alpha_n^- e^{-in\sigma^-}]. \quad (5.24)$$

X_{1B}^- is the same as that of our previous paper:¹⁾

$$X_{1B}^- = \mu\alpha' \sum_{n \in \mathbb{Z}} \left[\text{sgn}(n - \hat{\mu}) : A_n^\dagger A_n : - \text{sgn}(n + \hat{\mu}) : B_n^\dagger B_n : \right] \tau \\ - \frac{i\mu\alpha'}{2} \sum_{m \neq n} \frac{1}{m - n} \left[\frac{m + n - 2\hat{\mu}}{\omega_m^+ \omega_n^+} A_m^\dagger A_n e^{i(m-n)\sigma^+} - \frac{m + n + 2\hat{\mu}}{\omega_m^- \omega_n^-} B_m^\dagger B_n e^{i(m-n)\sigma^-} \right], \quad (5.25)$$

where we define $\omega_n^\pm = \sqrt{|n \mp \hat{\mu}|}$. In the summation with $m \neq n$ in Eq.(5.25), m and n run from $-\infty$ to ∞ , excluding $m = n$. Note that the terms in this summation are not influenced by the normal ordered product. From the above calculation (5.17)-(5.22), we can obtain the free mode representation of X_{1F}^- :

$$X_{1F}^- = \mu\alpha' \sum_{r \in \mathbb{Z} + \varepsilon} (: \tilde{\lambda}_r^\dagger \tilde{\lambda}_r : - : \lambda_r^\dagger \lambda_r :) \tau + i\mu\alpha' \sum_{\substack{n \neq 0 \\ r \in \mathbb{Z} + \varepsilon}} \frac{1}{n} [\tilde{\lambda}_r^\dagger \tilde{\lambda}_{n+r} e^{-in\sigma^+} - \lambda_r^\dagger \lambda_{n+r} e^{-in\sigma^-}]. \quad (5.26)$$

We can also represent X_{1B}^- and X_{1F}^- by using the fields f , g and λ_\pm :

$$X_{1B}^- = \frac{i\mu}{2} \left[\int d\sigma^+ : (f^* \partial_+ f - \partial_+ f^* f) : - \int d\sigma^- : (g^* \partial_- g - \partial_- g^* g) : \right] - \mu J_B \sigma, \quad (5.27)$$

$$X_{1F}^- = \frac{\mu}{2} \left[\int d\sigma^+ : \lambda_+^* \lambda_+ : - \int d\sigma^- : \lambda_-^* \lambda_- : \right] - \mu J_F \sigma. \quad (5.28)$$

Here the integrals are indefinite integrals, and we choose the constants of integration to be zero. Moreover J_B and J_F are

$$J_B = \frac{i}{4\pi} \left[\int_0^{2\pi} d\sigma^+ : (f^* \partial_+ f - \partial_+ f^* f) : + \int_0^{2\pi} d\sigma^- : (g^* \partial_- g - \partial_- g^* g) : \right], \quad (5.29)$$

$$J_F = \frac{1}{4\pi} \left[\int_0^{2\pi} d\sigma^+ : \lambda_+^* \lambda_+ : + \int_0^{2\pi} d\sigma^- : \lambda_-^* \lambda_- : \right]. \quad (5.30)$$

In the free-mode representation, J_B and J_F are given by

$$J_B = \alpha' \sum_{n \in \mathbb{Z}} \left[\text{sgn}(n - \hat{\mu}) : A_n^\dagger A_n : + \text{sgn}(n + \hat{\mu}) : B_n^\dagger B_n : \right], \quad (5.31)$$

$$J_F = \alpha' \sum_{r \in \mathbb{Z} + \varepsilon} \left[: \tilde{\lambda}_r^\dagger \tilde{\lambda}_r : + : \lambda_r^\dagger \lambda_r : \right]. \quad (5.32)$$

Substituting the free modes of f and g into X_2^- , it becomes

$$X_2^- = \frac{i\mu\alpha'}{2} \sum_{m,n \in \mathbb{Z}} \frac{1}{\omega_m^+ \omega_n^-} \left[A_m B_n^\dagger e^{-i(m-n-2\hat{\mu})\tau - i(m-n)\sigma} - A_m^\dagger B_n e^{i(m-n-2\hat{\mu})\tau + i(m-n)\sigma} \right]. \quad (5.33)$$

Therefore we can represent all the general operator solutions by using only free modes.

We now explicitly present the (anti)commutation relations for all the modes, in order to demonstrate that they are perfectly free-modes:

- The nonvanishing commutation relations between the modes of bosonic fields are

$$[x^+, p^-] = [x^-, p^+] = -i, \quad [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] = [\alpha_m^+, \alpha_n^-] = -m\delta_{m+n}, \quad (5.34)$$

$$[x^k, p^l] = i\delta^{kl}, \quad [\tilde{\alpha}_m^k, \tilde{\alpha}_n^l] = [\alpha_m^k, \alpha_n^l] = m\delta^{kl}\delta_{m+n}, \quad (5.35)$$

$$[A_m, A_n^\dagger] = \text{sgn}(m - \hat{\mu})\delta_{m,n}, \quad [B_m, B_n^\dagger] = \text{sgn}(m + \hat{\mu})\delta_{m,n}. \quad (5.36)$$

- The nonvanishing anticommutation relations between the modes of ψ_\pm^+ and $\psi_{0\pm}^-$ are

$$\{\tilde{\psi}_r^+, \tilde{\psi}_s^-\} = \{\psi_r^+, \psi_s^-\} = -\delta_{r+s}. \quad (5.37)$$

- The nonvanishing anticommutation relations between the modes of λ_\pm and λ_\pm^* are

$$\{\tilde{\lambda}_r, \tilde{\lambda}_s^\dagger\} = \{\lambda_r, \lambda_s^\dagger\} = \delta_{r-s}. \quad (5.38)$$

- The nonvanishing anticommutation relations between the modes of ψ_\pm^k are

$$\{\tilde{\psi}_r^k, \tilde{\psi}_s^l\} = \{\psi_r^k, \psi_s^l\} = \delta^{kl}\delta_{r-s}. \quad (5.39)$$

In this paper δ_{m+n} and $\delta_{r\pm s}$ denote the Kronecker delta, namely $\delta_{m+n,0}$ and $\delta_{r\pm s,0}$. All the other (anti)commutators between the modes vanish. We confirm in the next section that these

(anti)commutation relations of the modes are consistent with the canonical anticommutation relations of bosonic fields and fermionic fields.

Here we mention important notice. They are free mode representations but they are not what we call free field representations in the conformal field theory. Because x^- which is the zero mode of X_0^- does not commute the twist factor $\hat{\mu}$ which has the momentum p^+ , the commutation relations between the fields X_0^- and f, g, λ_{\pm} and their Hermitian conjugate fields do not vanish. We cannot remove these quantum interaction, however x^- is absent in the fields $\partial_{\pm} X_0^-$, so that $\partial_{\pm} X_0^-$ are just the same as free fields. Therefore we can calculate the super-Virasoro algebra without problem.

Finally we write up the definition of the normal orderings about the modes of f, g, λ_{\pm} .

- The normal orderings of $A_n^{\dagger} A_n$ and $B_n^{\dagger} B_n$ are

$$: A_n^{\dagger} A_n : = \begin{cases} A_n^{\dagger} A_n & (n > \hat{\mu}), \\ A_n A_n^{\dagger} & (n < \hat{\mu}), \end{cases} \quad (5.40)$$

$$: B_n^{\dagger} B_n : = \begin{cases} B_n^{\dagger} B_n & (n > -\hat{\mu}), \\ B_n B_n^{\dagger} & (n < -\hat{\mu}), \end{cases} \quad (5.41)$$

- The normal orderings of $\tilde{\lambda}_r^{\dagger} \tilde{\lambda}_r$ and $\lambda_r^{\dagger} \lambda_r$ are

$$: \tilde{\lambda}_r^{\dagger} \tilde{\lambda}_r : = \begin{cases} \tilde{\lambda}_r^{\dagger} \tilde{\lambda}_r & (r > \hat{\mu}), \\ -\tilde{\lambda}_r \tilde{\lambda}_r^{\dagger} & (r < \hat{\mu}), \end{cases} \quad (5.42)$$

$$: \lambda_r^{\dagger} \lambda_r : = \begin{cases} \lambda_r^{\dagger} \lambda_r & (r > -\hat{\mu}), \\ -\lambda_r \lambda_r^{\dagger} & (r < -\hat{\mu}), \end{cases} \quad (5.43)$$

Here, we assume that $\hat{\mu}$ is a real number excluding all integers and all half integers. Of course, the normal orderings of the modes $\tilde{\alpha}_n^{\pm}, \alpha_n^{\pm}, \tilde{\alpha}_n^k$ and α_n^k are exactly the same as that in the usual case of free fields. The normal ordering plays an important role in the calculation of the anomaly of the super-Virasoro algebra given in §7.

§6. Proof of the free-mode representation

In this section we prove that the general solutions appearing in §3 and the free-mode representations appearing in §5 satisfy the canonical (anti)commutation relations for all the covariant string coordinates and all the covariant fermionic fields. Because we need the canonical momentum to quantize the string coordinates, we obtain the canonical momentum from the action (2.14) using $P_{\mu} = \frac{\partial S_M}{\partial(\partial_{\tau} X^{\mu})}$, $P_Z = \frac{\partial S_M}{\partial(\partial_{\tau} Z)}$ and $P_{Z^*} = \frac{\partial S_M}{\partial(\partial_{\tau} Z^*)}$:

$$P_+ = -\frac{1}{2\pi\alpha'} \left[\partial_{\tau} X^- + \frac{i\mu}{2} (Z \partial_{\sigma} Z^* - Z^* \partial_{\sigma} Z) + \mu^2 \partial_{\tau} X^+ Z^* Z \right. \\ \left. - \frac{\mu}{2} (\psi_+^* \psi_+^Z - \psi_-^{Z*} \psi_-^Z) + \frac{i\mu^2}{2} \psi_+^+ (Z \psi_+^{Z*} + Z^* \psi_+^Z) + \frac{i\mu^2}{2} \psi_-^+ (Z \psi_-^{Z*} + Z^* \psi_-^Z) \right], \quad (6.1)$$

$$P_- = -\frac{1}{2\pi\alpha'}\partial_\tau X^+, \quad (6.2)$$

$$P_Z = \frac{1}{4\pi\alpha'}[\partial_\tau Z^* - i\mu\partial_\sigma X^+ Z^* + \mu(\psi_+^+ \psi_+^{Z*} - \psi_-^+ \psi_-^{Z*})], \quad (6.3)$$

$$P_{Z^*} = \frac{1}{4\pi\alpha'}[\partial_\tau Z + i\mu\partial_\sigma X^+ Z - \mu(\psi_+^+ \psi_+^Z - \psi_-^+ \psi_-^Z)], \quad (6.4)$$

$$P_k = \frac{1}{2\pi\alpha'}\partial_\tau X^k. \quad (6.5)$$

Here we note that it is not necessary to consider the momenta of the fermionic fields $\pi_{\mu A}$ by the reason from §4. Although the momentum appears complicated, it can be put into a simpler form by using the fields X^+ , \tilde{X}^+ , X_0^- , X^k , f , g , ψ_\pm^+ and λ_\pm which appear in the free-mode representation discussed in §5. The field P_+ becomes the most simplified:

$$P_+ = -\frac{1}{2\pi\alpha'}\partial_\tau X_0^-, \quad (6.6)$$

$$P_- = -\frac{1}{2\pi\alpha'}\partial_\tau X^+, \quad (6.7)$$

$$P_Z = \frac{1}{4\pi\alpha'}e^{i\mu\tilde{X}^+}[\partial_+ f^* + \partial_- g^* + \mu(\psi_+^+ \lambda_+^* - \psi_-^+ \lambda_-^*)], \quad (6.8)$$

$$P_{Z^*} = \frac{1}{4\pi\alpha'}e^{-i\mu\tilde{X}^+}[\partial_+ f + \partial_- g - \mu(\psi_+^+ \lambda_+ - \psi_-^+ \lambda_-)], \quad (6.9)$$

$$P_k = \frac{1}{2\pi\alpha'}\partial_\tau X^k. \quad (6.10)$$

Let us remember the canonical (anti)commutation relations (4.10)-(4.18) in §4 for quantization.

First, let us explain almost self-evident parts of the proof.

1. The proof of the canonical commutation relations between bosonic fields without X^- , P_Z and P_{Z^*} is the same as the proof in our previous paper.¹⁾ The reason why we remove X^- , P_Z and P_{Z^*} is that these fields contain fermionic fields. X^+ , X^k , and P_k are trivially free fields. so that these fields commute with all the other bosonic fields and fermionic fields. P_- commutes with all the fields without X_0^- in X^- . Therefore P_- and X^- satisfy the usual canonical commutation relation. Moreover Z and Z^* commute with all the fermionic fields. So they commute with the fermionic part of X^- , P_Z and P_{Z^*} . Thus Z , Z^* , X^- , P_Z and P_{Z^*} satisfy the usual canonical commutation relations, and moreover, P_+ commutes with X^- , P_Z , P_{Z^*} , ψ_A^Z , ψ_A^{Z*} and ψ_A^- because P_+ commutes with \tilde{X}^+ , in the light of the previous paper.¹⁾
2. The canonical commutation relations about $\pi_{\mu A}$ are the same as those of ψ_A^μ through the relation $\pi_{\mu A} = \frac{i}{4\pi\alpha'}G_{\mu\nu}\psi_A^\nu$. Because $G_{\mu\nu}$ is the function of X^μ , $G_{\mu\nu}$ commute with X^μ trivially. So the canonical commutation relations between $\pi_{\mu A}$ and X^μ are the same as the canonical commutation relations between ψ_A^μ and X^μ . Moreover we can calculate the commutation relations between $\pi_{\mu A}$ and P_μ using the Leibniz rule and the commutation relations between

P_μ and $G_{\mu\nu}$ whose commutation relation is calculated by the canonical commutation relations between X^μ and P_μ . Moreover $G_{\mu\nu}$ commute with ψ_A^μ . Thus we can prove the commutation relations about $\pi_{\mu A}$, using the canonical commutation relations about ψ_A^μ .

3. Clearly, ψ_A^k are the same as the ordinary free fermionic fields. Thus ψ_A^k satisfy the usual canonical anticommutation relations and ψ_A^k (anti)commute with all the other fields.
4. The commutation relations between ψ_A^+ and ψ_A^- are reduced to the commutation relation between ψ_A^+ and ψ_{0A}^- because ψ_A^+ (anti)commute with all the other fields.
5. We have constructed the free-mode representations from the canonical commutation relation between X^- and ψ_A^Z in §5, so that these relations and their Hermitian conjugate relations are satisfied trivially. Moreover X^- commutes with ψ_A^- because X^- commutes with Z , Z^* , ψ_A^Z , ψ_A^{Z*} and ψ_+^+ .

We present all the important parts of the proof in next subsections.

6.1. Canonical anticommutation relations between ψ_A^μ

In this subsection we prove the canonical anticommutation relations between ψ_A^μ . First we prove the canonical anticommutation relations between ψ_A^Z and ψ_A^{Z*} . Second we prove the canonical anticommutation relations between ψ_A^- and ψ_A^Z (ψ_A^{Z*}). Finally we prove the canonical anticommutation relations between ψ_A^- . The other anticommutation relations between ψ_A^μ have already been proved in the previous subsection.

6.1.1. Canonical commutation relations between ψ_A^Z and ψ_A^{Z*}

In the proof of the canonical anticommutation relations, we had better use the Eqs.(3-19) and (3-20) which are represented by using the fields Z and Z^* . Of course, we can also prove them by using the fields f and g . First we can prove that $\lambda_\pm(\sigma^\pm)$ commutes with ψ_\pm^+ , \tilde{X}^+ , Z and Z^* from the canonical anticommutation relations, so that the anticommutators between ψ_\pm^Z and ψ_\pm^{Z*} become the product of $e^{-i\mu[\tilde{X}^+(\sigma)-\tilde{X}^+(\sigma')]}$ and the simple anticommutators between λ_\pm and λ_\pm^* , namely the anticommutators $\{\psi_\pm^Z(\tau, \sigma), \psi_\pm^{Z*}(\tau, \sigma')\}$ are

$$\{\psi_\pm^Z(\tau, \sigma), \psi_\pm^{Z*}(\tau, \sigma')\} = e^{-i\mu[\tilde{X}^+(\sigma)-\tilde{X}^+(\sigma')]} \{\lambda_\pm(\sigma^+), \lambda_\pm^*(\sigma'^+)\}. \quad (6.11)$$

Let us pay attention to the case of the anticommutators between λ_+ and λ_+^* . Substituting the mode expansions of λ_+ and λ_+^* , this anticommutator becomes

$$\{\psi_+^Z(\tau, \sigma), \psi_+^{Z*}(\tau, \sigma')\} = 2\alpha' e^{-i\mu[\tilde{X}^+(\sigma)-\tilde{X}^+(\sigma')]} \sum_{r,s \in \mathbb{Z}+\varepsilon} \{\tilde{\lambda}_r, \tilde{\lambda}_s^\dagger\} e^{-i(r-\hat{\mu})\sigma^+} e^{i(s-\hat{\mu})\sigma'^+}$$

$$= 4\pi\alpha' e^{-i[\mu\tilde{X}^+(\sigma)-\hat{\mu}\sigma^+]} e^{i[\mu\tilde{X}^+(\sigma')-\hat{\mu}\sigma'^+]} \times \frac{1}{2\pi} \sum_{r \in \mathbb{Z}+\varepsilon} e^{-ir(\sigma-\sigma')}, \quad (6.12)$$

where $\{\tilde{\lambda}_r, \tilde{\lambda}_s^\dagger\} = \delta_{r-s}$, and the last term is corresponding to the delta-function. We must note that the anti-periodic delta-function arises when λ_+ and λ_+^* are the NS fermions. Using the delta-functions and the periodic function $F(\sigma) = -i[\mu\tilde{X}^+(\sigma) - \hat{\mu}\sigma^+]$, the anticommutator becomes

$$\{\psi_+^Z(\tau, \sigma), \psi_+^{Z*}(\tau, \sigma')\} = \begin{cases} 4\pi\alpha' e^{F(\sigma)-F(\sigma')} \delta(\sigma - \sigma') & : \text{R sector}, \\ 4\pi\alpha' e^{F(\sigma)-F(\sigma')} \tilde{\delta}(\sigma - \sigma') & : \text{NS sector}. \end{cases} \quad (6.13)$$

When we treat the case of the R sector, the function $e^{F(\sigma)-F(\sigma')}$ becomes 1 using the property of the periodic delta-function because this function is a periodic function of σ . When we treat the case of the NS sector, we use the property of the anti-periodic delta-function as follows:

$$[F(\sigma) - F(\sigma')] \tilde{\delta}(\sigma - \sigma') = 0, \quad (6.14)$$

where we can prove this relation (6.14) by using the Fourier expansion. Moreover Taylor expansion of the function $e^{F(\sigma)-F(\sigma')}$ is $1 + \sum_{n=1}^{\infty} \frac{1}{n!} [F(\sigma) - F(\sigma')]^n$, so that this function also becomes 1 in the case of the NS sector of Eq.(6.13). Therefore the canonical anticommutation relation (4.14) is proved, because we can also prove the case of ψ_-^Z and ψ_-^{Z*} using the same procedure.

6.1.2. Canonical anticommutation relations between ψ_\pm^Z (ψ_\pm^{Z*}) and ψ_\pm^-

First we prove the canonical anticommutator relation $\{\psi_\pm^Z(\tau, \sigma), \psi_\pm^-(\tau, \sigma')\} = 0$. Using the general solutions (3.19), (3.27), (3.28) and the canonical anticommutation relations (4.13), (4.14), the anticommutator are reduced to

$$\begin{aligned} \{\psi_\pm^Z(\tau, \sigma), \psi_\pm^-(\tau, \sigma')\} &= \mp i\mu \{\psi_\pm^+(\sigma^\pm), \psi_\pm^-(\tau, \sigma')\} Z(\sigma) \mp \frac{i}{2} \mu Z(\sigma') \{\psi_\pm^Z(\tau, \sigma), \psi_\pm^{Z*}(\tau, \sigma')\} \\ &= \begin{cases} \pm 2\pi i \alpha' [Z(\sigma) - Z(\sigma')] \delta(\sigma - \sigma') & : \text{R sector}, \\ \pm 2\pi i \alpha' [Z(\sigma) - Z(\sigma')] \tilde{\delta}(\sigma - \sigma') & : \text{NS sector}. \end{cases} \end{aligned} \quad (6.15)$$

Here $Z(\sigma)$ is the periodic function of σ , so that we can use the relation (6.14) in the NS sector again. The usual property of the periodic delta-function can also be used in the R sector. Therefore the anticommutators become zero. We can prove the case of ψ_\pm^{Z*} using the same procedure or taking the Hermitian conjugate of ψ_\pm^Z .

In this proof, we can obtain the useful formulae from Eq.(6.15). First, from the canonical anticommutation relations between $\psi_\pm^-(\tau, \sigma)$ and $\psi_\pm^+(\sigma'^\pm)$, the following relations are given:

$$\{\psi_{0\pm}^-(\sigma^\pm), \psi_\pm^+(\sigma'^\pm)\} = \begin{cases} -2\pi\alpha' \delta(\sigma - \sigma') & : \text{R sector}, \\ -2\pi\alpha' \tilde{\delta}(\sigma - \sigma') & : \text{NS sector}, \end{cases} \quad (6.16)$$

because ψ_{\pm}^{\pm} (anti)commute with all the other fields. Second, from the canonical anticommutation relations between $\psi_{\pm}^Z(\tau, \sigma)$, $\psi_{\pm}^{Z*}(\tau, \sigma)$ and $\psi_{\pm}^{-}(\tau, \sigma')$, the following relations are given:

$$\{\psi_{\pm}^Z(\tau, \sigma), \psi_{0\pm}^{-}(\sigma'^{\pm})\} = \begin{cases} \pm i2\pi\alpha' \mu Z \delta(\sigma - \sigma') & : \text{R sector}, \\ \pm i2\pi\alpha' \mu Z \tilde{\delta}(\sigma - \sigma') & : \text{NS sector}, \end{cases} \quad (6.17)$$

$$\{\psi_{\pm}^{Z*}(\tau, \sigma), \psi_{0\pm}^{-}(\sigma'^{\pm})\} = \begin{cases} \mp i2\pi\alpha' \mu Z^* \delta(\sigma - \sigma') & : \text{R sector}, \\ \mp i2\pi\alpha' \mu Z^* \tilde{\delta}(\sigma - \sigma') & : \text{NS sector}, \end{cases} \quad (6.18)$$

because the fields ψ_{\pm}^Z and ψ_{\pm}^{Z*} have ψ_{\pm}^{\pm} which do not anticommute with $\psi_{0\pm}^{-}$. These formulae are used in the next subsection (6.1.3).

6.1.3. Canonical anticommutation relations between ψ_A^{-}

These anticommutation relations, which are shown in the Eq.(4.12), are not zero in the pp -wave background. Let us calculate the anticommutator $\{\psi_{\pm}^{-}(\tau, \sigma), \psi_{\pm}^{-}(\tau, \sigma')\}$, where the general operator solutions of ψ_{\pm}^{-} are presented in Eqs.(3.27) and (3.28). The free field $\psi_{0\pm}^{-}$ (anti)commute with all the fields without ψ_{\pm}^{\pm} , ψ_{\pm}^Z and ψ_{\pm}^{Z*} . Thus the following fields survive in the anticommutators $\{\psi_{\pm}^{-}(\tau, \sigma), \psi_{\pm}^{-}(\tau, \sigma')\}$:

$$\begin{aligned} \{\psi_{\pm}^{-}(\tau, \sigma), \psi_{\pm}^{-}(\tau, \sigma')\} = & \pm \frac{i}{2} \mu Z(\sigma') \{\psi_{0\pm}^{-}(\sigma^{\pm}), \psi_{\pm}^{Z*}(\sigma')\} \pm \frac{i}{2} \mu Z^*(\sigma') \{\psi_{0\pm}^{-}(\sigma^{\pm}), \psi_{\pm}^Z(\sigma')\} \\ & - \mu^2 \{\psi_{0\pm}^{-}(\sigma^{\pm}), \psi_{\pm}^{\pm}(\sigma'^{\pm})\} Z^*(\sigma') Z(\sigma') \\ & \mp \frac{i}{2} \mu Z(\sigma) \{\psi_{\pm}^{Z*}(\sigma), \psi_{\pm}^{-}(\sigma'^{\pm})\} + \frac{1}{4} \mu^2 Z(\sigma) Z^*(\sigma') \{\psi_{\pm}^{Z*}(\sigma), \psi_{\pm}^Z(\sigma')\} \\ & \pm \frac{i}{2} \mu Z^*(\sigma) \{\psi_{\pm}^Z(\sigma), \psi_{\pm}^{-}(\sigma'^{\pm})\} + \frac{1}{4} \mu^2 Z^*(\sigma) Z(\sigma') \{\psi_{\pm}^Z(\sigma), \psi_{\pm}^{Z*}(\sigma')\} \\ & - \mu^2 \{\psi_{\pm}^{\pm}(\sigma^{\pm}), \psi_{0\pm}^{-}(\sigma'^{\pm})\} Z^*(\sigma) Z(\sigma). \end{aligned} \quad (6.19)$$

Using the useful formulae (6.16)-(6.18), we can reproduce the canonical anticommutation relation (4.12).

We have to note that we can also identify the world-sheet coordinates of the periodic function $F(\sigma)$ in the case of anti-periodic delta-function $\tilde{\delta}(\sigma - \sigma')$ from the relation (6.14). Thus $F(\sigma') \tilde{\delta}(\sigma - \sigma') = F(\sigma) \tilde{\delta}(\sigma - \sigma')$. Moreover we have to take notice of $\{\psi_{0\pm}^{-}(\sigma^{\pm}), \psi_{0\pm}^{-}(\sigma'^{\pm})\} = 0$, which means that $\psi_{0\pm}^{-}$ are just free fields.

6.2. Canonical commutation relations between P_{μ} and ψ_A^{μ}

In this subsection we prove the canonical commutation relations between P_{μ} and ψ_A^{μ} . First we prove the canonical commutation relations between P_Z (P_{Z^*}) and ψ_{\pm}^Z (ψ_{\pm}^{Z*}). Second we prove the canonical commutation relations between P_Z , P_{Z^*} and ψ_{\pm}^{-} . The other canonical commutation relations between P_{μ} and ψ_A^{μ} have already been proved in the previous subsection.

6.2.1. Canonical commutation relations between P_Z (P_{Z^*}) and ψ_A^Z ($\psi_A^{Z^*}$)

We calculate the commutation relations $[P_Z(\tau, \sigma), \psi_\pm^Z(\tau, \sigma')]$. First we divide P_Z (P_{Z^*}) into the bosonic part P_Z^B ($P_{Z^*}^B$) and the fermionic part P_Z^F ($P_{Z^*}^F$) as follows:

$$P_Z = P_Z^B + P_Z^F, \quad (6.20)$$

$$P_Z^B = \frac{1}{4\pi\alpha'} [\partial_+ Z^* + \partial_- Z^* - i\mu(\partial_+ X^+ - \partial_- X^+) Z^*], \quad (6.21)$$

$$P_Z^F = \frac{\mu}{4\pi\alpha'} [\psi_+^+ \psi_+^{Z^*} - \psi_-^+ \psi_-^{Z^*}], \quad (6.22)$$

where P_{Z^*} is obtained by Hermitian conjugate of P_Z . Therefore the commutators between P_Z and ψ_\pm^Z become as follows:

$$[P_Z(\tau, \sigma), \psi_\pm^Z(\tau, \sigma')] = [P_Z^B(\sigma) + P_Z^F(\sigma), \psi_\pm^Z(\sigma')] = [P_Z^B(\sigma), \psi_\pm^Z(\sigma')] + [P_Z^F(\sigma), \psi_\pm^Z(\sigma')]. \quad (6.23)$$

We have to note that ψ_\pm^Z have the field Z in Eq. (3.19), so that the commutation relations become as follows:

$$\begin{aligned} [P_Z^B(\tau, \sigma), \psi_\pm^Z(\tau, \sigma')] &= \mp i\mu\psi_\pm^+(\sigma'^{\pm})[P_Z^B(\sigma), Z(\sigma')] \\ &= \mp \mu\psi_\pm^+(\sigma'^{\pm})\delta(\sigma - \sigma'). \end{aligned} \quad (6.24)$$

Moreover the commutation relations between P_Z^F and ψ_\pm^Z become as follows:

$$[P_Z^F(\sigma), \psi_\pm^Z(\sigma')] = \begin{cases} \pm \mu\psi_\pm^+(\sigma^{\pm})\delta(\sigma - \sigma') & : \text{R sector}, \\ \pm \mu\psi_\pm^+(\sigma^{\pm})\tilde{\delta}(\sigma - \sigma') & : \text{NS sector}. \end{cases} \quad (6.25)$$

In the case of the R sector, we can identify the world-sheet coordinates, so that the commutator (6.23) vanish. In the case of the NS sector, using the relation (4.9), the commutator (6.24) become as follows:

$$[P_Z^B(\tau, \sigma), \psi_\pm^Z(\tau, \sigma')] = \mp \mu\psi_\pm^+(\sigma^{\pm})\tilde{\delta}(\sigma - \sigma'). \quad (6.26)$$

Therefore the commutators (6.23) also vanish in the NS sector, so that the canonical commutation relation $[P_Z^B(\tau, \sigma), \psi_\pm^Z(\tau, \sigma')] = 0$ is proved.

6.2.2. Canonical commutation relations between P_Z , P_{Z^*} and ψ_A^-

Let us pay attention to these commutation relations which are not zero in the pp-wave background. We calculate the commutator $[P_Z(\tau, \sigma), \psi_\pm^-(\tau, \sigma')]$. Here P_Z^B commute with Z^* and $\psi_\pm^{Z^*}$, and the commutators between P_Z^B and ψ_\pm^Z are written in Eq. (6.24), and we have to note that P_Z^F dose not commute with ψ_\pm^- , so that the commutators are reduced to

$$\begin{aligned} [P_Z(\tau, \sigma), \psi_\pm^-(\tau, \sigma')] &= \mp \frac{i}{2}\mu[P_Z^B(\sigma), Z(\sigma')]\psi_\pm^{Z^*}(\sigma') \pm \frac{i}{2}\mu Z^*(\sigma')[P_Z^B(\sigma), \psi_\pm^Z(\sigma')] \\ &\quad - \mu^2\psi_\pm^+ Z^*(\sigma')[P_Z^B(\sigma), Z(\sigma')] + [P_Z^F(\sigma), \psi_\pm^-(\sigma')]. \end{aligned} \quad (6.27)$$

Here the commutators $[P_Z^F(\sigma), \psi_\pm^-(\sigma')]$ are calculated as

$$[P_Z^F(\sigma), \psi_\pm^-(\sigma')] = \mp \frac{\mu}{4\pi\alpha'} \psi_\pm^Z(\sigma) \{\psi_\pm^+(\sigma), \psi_\pm^-(\sigma')\}. \quad (6.28)$$

Thus the commutators become the following form:

$$[P_Z(\tau, \sigma), \psi_\pm^-(\tau, \sigma')] = \left(\mp \frac{i}{2} \mu \psi_\pm^{Z*}(\sigma') - \frac{\mu^2}{2} Z^* \psi_\pm^+(\sigma') \right) [P_Z^B(\sigma), Z(\sigma')] \mp \frac{\mu}{4\pi\alpha'} \psi_\pm^Z(\sigma) \{\psi_\pm^+(\sigma), \psi_\pm^-(\sigma')\} \quad (6.29)$$

Here we note that the delta-function from $[P_Z(\sigma), Z(\sigma')]$ is the periodic delta-function, and in the case of the R fermions, we obtain the periodic delta-function $\{\psi_\pm^+(\sigma), \psi_\pm^-(\sigma')\} = -2\pi\alpha' \delta(\sigma - \sigma')$. So we can calculate it easily and we obtain the canonical commutation relation. In the case of the NS fermions, we can obtain the anti-periodic delta-function $\{\psi_\pm^+(\sigma), \psi_\pm^-(\sigma')\} = -2\pi\alpha' \tilde{\delta}(\sigma - \sigma')$, and we use the identity (4.9) for the NS fermions. Therefore we obtain the same result as the R fermions, so that we can confirm the canonical commutation relation (4.17). We can also prove the commutation relation (4.18) using the same procedure.

6.3. Canonical commutation relations between the bosonic fields

In this subsection we prove the commutation relations between the bosonic fields and their momenta, in particular X^- , P_Z and P_{Z^*} which have fermionic fields. First we prove the commutation relation between P_Z and P_{Z^*} . Second we prove the commutation relations between X^- and X^- . Third we prove the commutation relation between X^- and P_Z (P_{Z^*}). The other commutation relations between the bosonic fields and their momenta have already been proved in the previous subsection.

6.3.1. Canonical commutation relation between P_Z and P_{Z^*}

We prove $[P_Z(\tau, \sigma), P_{Z^*}(\tau, \sigma')] = 0$. Since P_Z and P_{Z^*} contain the fermionic fields, we divide P_Z (P_{Z^*}) into the bosonic part P_Z^B ($P_{Z^*}^B$) and the fermionic part P_Z^F ($P_{Z^*}^F$). Therefore the commutator is

$$\begin{aligned} [P_Z(\tau, \sigma), P_{Z^*}(\tau, \sigma')] &= [P_Z^B(\sigma) + P_Z^F(\sigma), P_{Z^*}^B(\sigma') + P_{Z^*}^F(\sigma')] \\ &= [P_Z^B(\sigma), P_{Z^*}^F(\sigma')] + [P_Z^F(\sigma), P_{Z^*}^B(\sigma')] + [P_Z^F(\sigma), P_{Z^*}^F(\sigma')]. \end{aligned} \quad (6.30)$$

Here we proved $[P_Z^B(\sigma), P_{Z^*}^B(\sigma')] = 0$ exactly in the previous paper.¹⁾ So we have only to prove $[P_Z^F(\tau, \sigma), P_{Z^*}^F(\tau, \sigma')] = 0$ because the commutator $[P_Z^B(\sigma), P_{Z^*}^F(\sigma')]$ and $[P_Z^F(\sigma), P_{Z^*}^B(\sigma')]$ are trivially zero. The commutation relation $[P_Z^F(\tau, \sigma), P_{Z^*}^F(\tau, \sigma')]$ becomes as follows:

$$[P_Z^F(\tau, \sigma), P_{Z^*}^F(\tau, \sigma')] = - \left(\frac{\mu}{4\pi\alpha'} \right)^2 ([\psi_+^+(\sigma) \psi_+^{Z*}(\sigma), \psi_+^+(\sigma') \psi_+^Z(\sigma')] + [\psi_-^+(\sigma) \psi_-^{Z*}(\sigma), \psi_-^+(\sigma') \psi_-^Z(\sigma')]) . \quad (6.31)$$

ψ_{\pm}^{\pm} anticommute with all fermionic fields other than $\psi_{0\pm}^{\pm}$, so that the commutator is reduced to the following:

$$[P_Z^F(\tau, \sigma), P_{Z^*}^F(\tau, \sigma)] = \left(\frac{\mu}{4\pi\alpha'} \right)^2 (\psi_+^+(\sigma)\psi_+^+(\sigma')\{\psi_+^{Z*}(\sigma), \psi_+^{Z*}(\sigma')\} + \psi_-^+(\sigma)\psi_-^+(\sigma')\{\psi_-^{Z*}(\sigma), \psi_-^{Z*}(\sigma')\}) , \quad (6.32)$$

where $\{\psi_{\pm}^{Z*}(\sigma), \psi_{\pm}^{Z*}(\sigma')\} = 4\pi\alpha'\delta(\sigma - \sigma')$ in the R sector and $\{\psi_{\pm}^{Z*}(\sigma), \psi_{\pm}^{Z*}(\sigma')\} = 4\pi\alpha'\tilde{\delta}(\sigma - \sigma')$ in the NS sector. Using the relation (4.9), we can make the periodic delta-function even in the NS sector, so that we identify the world-sheet coordinates of $\psi_{\pm}^+(\sigma)\psi_{\pm}^+(\sigma')$. From the canonical anti-commutation relations $\{\psi_{\pm}^+(\sigma), \psi_{\pm}^+(\sigma')\} = 0$, $(\psi_{\pm}^+)^2$ vanishes. Therefore the commutation relation $[P_Z(\tau, \sigma), P_{Z^*}(\tau, \sigma')] = 0$ is proved.

6.3.2. Canonical commutation relation between X^- and X^-

We calculate the commutator $[X^-(\tau, \sigma), X^-(\tau, \sigma')]$. Here we define X_B^- , which is the bosonic part of X^- , as $X_0^- + X_{1B}^- + X_2^-$. We proved $[X_B^-(\tau, \sigma), X_B^-(\tau, \sigma')] = 0$ exactly in the previous paper¹⁾ and $[X_B^-(\tau, \sigma), X_{1F}^-(\tau, \sigma')]$ is trivially zero, so that we have only to prove $[X_{1F}^-(\tau, \sigma), X_{1F}^-(\tau, \sigma')] = 0$, where $X_{1F}^-(\tau, \sigma)$ is defined in (5.26). We have only to prove the case of $\tau = 0$ because we can describe the time evolution using the Hamiltonian of the system in the same method of the previous paper.¹⁾ The commutator becomes the following:

$$\begin{aligned} & [X_{1F}^-(\sigma), X_{1F}^-(\sigma')]|_{\tau=0} \\ &= -\mu^2\alpha'^2 \sum_{m,n \neq 0} \sum_{r,s \in Z+\epsilon} \frac{1}{mn} [\tilde{\lambda}_r^\dagger \tilde{\lambda}_{m+r} e^{-im\sigma} - \lambda_r^\dagger \lambda_{m+r} e^{im\sigma}, \tilde{\lambda}_s^\dagger \tilde{\lambda}_{n+s} e^{-in\sigma'} - \lambda_s^\dagger \lambda_{n+s} e^{in\sigma'}] \\ &= -\mu^2\alpha'^2 \sum_{m,n \neq 0} \sum_{r,s \in Z+\epsilon} \frac{1}{mn} \left([\tilde{\lambda}_r^\dagger \tilde{\lambda}_{m+r}, \tilde{\lambda}_s^\dagger \tilde{\lambda}_{n+s}] e^{-im\sigma - in\sigma'} + [\lambda_r^\dagger \lambda_{m+r}, \lambda_s^\dagger \lambda_{n+s}] e^{im\sigma + in\sigma'} \right). \quad (6.33) \end{aligned}$$

Here we use the formula of commutation relation of Grassmann numbers:

$$[\alpha\beta, \gamma\delta] = \alpha\{\beta, \gamma\}\delta - \{\alpha, \gamma\}\beta\delta + \gamma\alpha\{\beta, \delta\} - \gamma\{\alpha, \delta\}\beta, \quad (6.34)$$

where $\alpha, \beta, \gamma, \delta$ are arbitrary Grassmann numbers. Using this formula, the commutator becomes

$$\begin{aligned} \sum_{r,s \in Z+\epsilon} [\lambda_r^\dagger \lambda_{m+r}, \lambda_s^\dagger \lambda_{n+s}] &= \sum_{r,s \in Z+\epsilon} \left(\lambda_r^\dagger \lambda_{n+s} \delta_{r-s+m} - \lambda_s^\dagger \lambda_{m+r} \delta_{s-r+n} \right) \\ &= \sum_{r \in Z+\epsilon} \left(\lambda_r^\dagger \lambda_{n+m+r} - \lambda_{r-n}^\dagger \lambda_{r+m} \right). \quad (6.35) \end{aligned}$$

After carrying out the summation over s , we change the suffix as $r \rightarrow r - n$ in the first term in Eq.(6.35), so that this commutator vanish. We can prove the case of $\tilde{\lambda}_r$ in the same method. Of course we can prove the case of $\tau \neq 0$. Therefore $[X^-(\tau, \sigma), X^-(\tau, \sigma')] = 0$ is proved.

6.3.3. Canonical commutation relation between X^- and P_Z (P_{Z*})

We prove the commutation relation $[X^-(\tau, \sigma), P_Z(\tau, \sigma')] = 0$. We can divide X^- into X_B^- and X_{1F}^- . and moreover, we can also divide P_Z into P_Z^B and P_Z^F . We proved the commutation relation between X_B^- and P_Z^B in the previous paper,¹⁾ and the commutation relation between X_{1F}^- and P_Z^B is trivially zero. So we have only to prove the commutation relation between X^- and P_Z^F .

$$[X^-(\tau, \sigma), P_Z(\tau, \sigma')] = [X^-(\tau, \sigma), P_Z^F(\tau, \sigma')] \quad (6.36)$$

where P_Z^F consists of ψ_\pm^+ and ψ_\pm^{Z*} . ψ_\pm^+ commute with X^- trivially, and we have already proved the canonical commutation relation between X^- and ψ_\pm^{Z*} in the previous section. Therefore the commutation relation between X^- and P_Z is proved. We can also prove the case of P_{Z*} using the same methods.

We have thus completed the proofs of all the equal-time canonical (anti)commutation relations in the free-mode representations.

§7. Super-Virasoro algebra and anomaly

In this section we define the energy-momentum tensor, the supercurrent and the super-Virasoro generators using the normal procedure. It is characteristic that they are represented as almost free cases in using our general operator solutions and our free mode representation. Therefore, we can exactly calculate the commutators or the anticommutators between the super-Virasoro generators and obtain the super-Virasoro anomaly.

The energy momentum tensor of the matter $T_{\alpha\beta}^M$ is defined as the response to variations of the world-sheet zweibein in the action, and the supercurrent of the matter $T_{F\alpha A}^M$ is similarly defined as the response to variations of the world-sheet gravitino in the action (2.1) :

$$T_{\alpha\beta}^M = -\frac{2\pi}{e} \frac{\delta S_M}{\delta e_a^\alpha} e_{\beta a} \ , \quad T_{F\alpha A}^M = -\frac{\sqrt{2}\pi i}{e} \frac{\delta S_M}{\delta \bar{\chi}^{\alpha A}} \ , \quad (7.1)$$

where A is the spinor component, and the variation of gravitino is performed from the right-hand side. Due to the tracelessness of the energy momentum tensor and the supercurrent, the only non-vanishing components are $T_{\pm\pm}^M$ and $T_{F\pm\pm}^M$. Here, the energy momentum tensor $T_{\pm\pm}^M$ are written in the world-sheet light-cone coordinates (σ^\pm) system according to the rules of tensor analysis, and $_{++}$ ($_{--}$) in the supercurrent T_{F++}^M (T_{F--}^M) denotes the σ^+ (σ^-) vector component and the upper (lower) spinor component. We fix the covariant gauge as $e_\alpha^a = \delta_\alpha^a$ and $\chi_\alpha = 0$ after the variation of the action. Therefore the energy-momentum tensor and the supercurrent of the matter in the

pp -wave background with the NS-NS flux are

$$T_{++}^M = \frac{1}{\alpha'} \left[-2\partial_+ X^+ \partial_+ X^- - \mu^2 Z^* Z \partial_+ X^+ \partial_+ X^+ + \partial_+ Z^* \partial_+ Z + \partial_+ X^k \partial_+ X^k \right. \\ \left. + \frac{i}{2} \left\{ -\psi_+^+ \partial_+ \psi_+^- - \psi_+^- \partial_+ \psi_+^+ - \mu^2 Z^* Z \psi_+^+ \partial_+ \psi_+^+ + \frac{1}{2} \psi_+^Z \partial_+ \psi_+^{Z*} + \frac{1}{2} \psi_+^{Z*} \partial_+ \psi_+^Z \right. \right. \\ \left. - \mu^2 \partial_+ X^+ Z^* \psi_+^+ \psi_+^Z - \mu^2 \partial_+ X^+ Z \psi_+^+ \psi_+^{Z*} - i\mu \partial_+ Z \psi_+^+ \psi_+^{Z*} + i\mu \partial_+ Z^* \psi_+^+ \psi_+^Z \right. \\ \left. + \frac{i}{2} \mu \partial_+ X^+ \psi_+^Z \psi_+^{Z*} - \frac{i}{2} \mu \partial_+ X^+ \psi_+^{Z*} \psi_+^Z + \psi_+^k \partial_+ \psi_+^k \right\} \right] , \quad (7.2)$$

$$T_{--}^M = \frac{1}{\alpha'} \left[-2\partial_- X^+ \partial_- X^- - \mu^2 Z^* Z \partial_- X^+ \partial_- X^+ + \partial_- Z^* \partial_- Z + \partial_- X^k \partial_- X^k \right. \\ \left. + \frac{i}{2} \left\{ -\psi_-^+ \partial_- \psi_-^- - \psi_-^- \partial_- \psi_-^+ - \mu^2 Z^* Z \psi_-^+ \partial_- \psi_-^+ + \frac{1}{2} \psi_-^Z \partial_- \psi_-^{Z*} + \frac{1}{2} \psi_-^{Z*} \partial_- \psi_-^Z \right. \right. \\ \left. - \mu^2 \partial_- X^+ Z^* \psi_-^+ \psi_-^Z - \mu^2 \partial_- X^+ Z \psi_-^+ \psi_-^{Z*} + i\mu \partial_- Z \psi_-^+ \psi_-^{Z*} - i\mu \partial_- Z^* \psi_-^+ \psi_-^Z \right. \\ \left. - \frac{i}{2} \mu \partial_- X^+ \psi_-^Z \psi_-^{Z*} + \frac{i}{2} \mu \partial_- X^+ \psi_-^{Z*} \psi_-^Z + \psi_-^k \partial_- \psi_-^k \right\} \right] , \quad (7.3)$$

$$T_{F++}^M = \frac{\sqrt{2}}{\alpha'} \left[-\psi_+^+ \partial_+ X^- - \psi_+^- \partial_+ X^+ - \mu^2 Z^* Z \psi_+^+ \partial_+ X^+ \right. \\ \left. + \frac{1}{2} \psi_+^Z \partial_+ Z^* + \frac{1}{2} \psi_+^{Z*} \partial_+ Z + \psi_+^k \partial_+ X^k - \frac{1}{2} \mu \psi_+^+ \psi_+^Z \psi_+^{Z*} \right] , \quad (7.4)$$

$$T_{F--}^M = \frac{\sqrt{2}}{\alpha'} \left[-\psi_-^+ \partial_- X^- - \psi_-^- \partial_- X^+ - \mu^2 Z^* Z \psi_-^+ \partial_- X^+ \right. \\ \left. + \frac{1}{2} \psi_-^Z \partial_- Z^* + \frac{1}{2} \psi_-^{Z*} \partial_- Z + \psi_-^k \partial_- X^k + \frac{1}{2} \mu \psi_-^+ \psi_-^Z \psi_-^{Z*} \right] . \quad (7.5)$$

Moreover, substituting the general solutions of the matter, (3.9)-(3.13), (3.15), (3.16), (3.19), (3.20), (3.29), (3.30) and (5.23), into the energy-momentum tensor and the supercurrent, they become

$$T_{++}^M = \frac{1}{\alpha'} \left[-2 : \partial_+ X^+ \partial_+ X_0^- : + \mu \partial_+ X^+ (J_B + J_F) + : \partial_+ f^* \partial_+ f : + : \partial_+ X^k \partial_+ X^k : \right. \\ \left. + \frac{i}{2} \left\{ - : \psi_+^+ \partial_+ \psi_{0+}^- : - : \psi_{0+}^- \partial_+ \psi_+^+ : + \frac{1}{2} : \lambda_+ \partial_+ \lambda_+^* : + \frac{1}{2} : \lambda_+^* \partial_+ \lambda_+ : + : \psi_+^k \partial_+ \psi_+^k : \right\} \right] , \quad (7.6)$$

$$T_{--}^M = \frac{1}{\alpha'} \left[-2 : \partial_- X^+ \partial_- X_0^- : - \mu \partial_- X^+ (J_B + J_F) + : \partial_- g^* \partial_- g : + : \partial_- X^k \partial_- X^k : \right. \\ \left. + \frac{i}{2} \left\{ - : \psi_-^+ \partial_- \psi_{0-}^- : - : \psi_{0-}^- \partial_- \psi_-^+ : + \frac{1}{2} : \lambda_- \partial_- \lambda_-^* : + \frac{1}{2} : \lambda_-^* \partial_- \lambda_- : + : \psi_-^k \partial_- \psi_-^k : \right\} \right] , \quad (7.7)$$

$$T_{F++}^M = \frac{\sqrt{2}}{\alpha'} \left[-\psi_+^+ \partial_+ X_+^- - \psi_{0+}^- \partial_+ X^+ + \frac{1}{2} \mu \psi_+^+ (J_B + J_F) + \frac{1}{2} \lambda_+ \partial_+ f^* + \frac{1}{2} \lambda_+^* \partial_+ f + \psi_+^k \partial_+ X_+^k \right] , \quad (7.8)$$

$$T_{F--}^M = \frac{\sqrt{2}}{\alpha'} \left[-\psi_-^+ \partial_- X_-^- - \psi_{0-}^- \partial_- X^+ - \frac{1}{2} \mu \psi_-^+ (J_B + J_F) + \frac{1}{2} \lambda_- \partial_- g^* + \frac{1}{2} \lambda_-^* \partial_- g + \psi_-^k \partial_- X_-^k \right] . \quad (7.9)$$

Although the interaction remains in the terms $\partial_\pm X^+ (J_B + J_F)$ and $\psi_\pm^\pm (J_B + J_F)$, we can completely calculate the anomalies of the super-Virasoro algebra in the same manner as the free fields.

Next, we define the super-Virasoro generators, which are the Fourier coefficients of the energy-momentum tensor and the supercurrent :

$$\tilde{L}_n^M = \int_0^{2\pi} \frac{d\sigma}{2\pi} e^{in\sigma} T_{++}^M, \quad L_n^M = \int_0^{2\pi} \frac{d\sigma}{2\pi} e^{-in\sigma} T_{--}^M, \quad (7.10)$$

$$\tilde{G}_r^M = \int_0^{2\pi} \frac{d\sigma}{2\pi} e^{ir\sigma} T_{F++}^M, \quad G_r^M = \int_0^{2\pi} \frac{d\sigma}{2\pi} e^{-ir\sigma} T_{F--}^M. \quad (7.11)$$

The generator \tilde{L}_n^M (L_n^M) can be divided into $\tilde{L}_n^{(+k)}$ ($L_n^{(+k)}$), which is a part of X^+ , X_0^- , X^k , ψ^+ , ψ_0^- , ψ^k , J_B and J_F , \tilde{L}_n^f (L_n^g), which is a part of f (g), and \tilde{L}_n^λ (L_n^λ), which is a part of λ_+ (λ_-), as $\tilde{L}_n^M = \tilde{L}_n^{(+k)} + \tilde{L}_n^f + \tilde{L}_n^\lambda$ and $L_n^M = L_n^{(+k)} + L_n^g + L_n^\lambda$. We can analogously divide the generator \tilde{G}_r^M (G_r^M) into $\tilde{G}_r^{(+k)}$ ($G_r^{(+k)}$), which is a part of X^+ , X_0^- , X^k , ψ^+ , ψ_0^- , ψ^k , J_B and J_F , $\tilde{G}_r^{\lambda f}$ ($G_r^{\lambda g}$), which is a part of f and λ_+ (g and λ_-), as $\tilde{G}_r^M = \tilde{G}_r^{(+k)} + \tilde{G}_r^{\lambda f}$ and $G_r^M = G_r^{(+k)} + G_r^{\lambda g}$. Moreover in our free-mode representation, the super-Virasoro generators are as follows.

- $\tilde{L}_n^{(+k)}$ and $L_n^{(+k)}$ in the super-Virasoro generators are

$$\begin{aligned} \tilde{L}_n^{(+k)} &= \frac{1}{2} \sum_{m \in \mathbb{Z}} [-2 : \tilde{\alpha}_{n-m}^+ \tilde{\alpha}_m^- : + : \tilde{\alpha}_{n-m}^k \tilde{\alpha}_m^k :] \\ &+ \frac{1}{2} \sum_{r \in \mathbb{Z} + \varepsilon} [-2(r - \frac{n}{2}) : \tilde{\psi}_{n-r}^+ \tilde{\psi}_r^- : + (r - \frac{n}{2}) : \tilde{\psi}_{n-r}^k \tilde{\psi}_r^k :] \\ &+ \frac{\mu}{\sqrt{2\alpha'}} \tilde{\alpha}_n^+ (J_B + J_F), \end{aligned} \quad (7.12)$$

$$\begin{aligned} L_n^{(+k)} &= \frac{1}{2} \sum_{m \in \mathbb{Z}} [-2 : \alpha_{n-m}^+ \alpha_m^- : + : \alpha_{n-m}^k \alpha_m^k :] \\ &+ \frac{1}{2} \sum_{r \in \mathbb{Z} + \varepsilon} [-2(r - \frac{n}{2}) : \psi_{n-r}^+ \psi_r^- : + (r - \frac{n}{2}) : \psi_{n-r}^k \psi_r^k :] \\ &- \frac{\mu}{\sqrt{2\alpha'}} \alpha_n^+ (J_B + J_F). \end{aligned} \quad (7.13)$$

- \tilde{L}_n^f , L_n^g , \tilde{L}_n^λ , and L_n^λ in the super-Virasoro generators are

$$\tilde{L}_n^f = \sum_{m \in \mathbb{Z}} (m - n - \hat{\mu})(m - \hat{\mu}) : \hat{A}_{m-n}^\dagger \hat{A}_m :, \quad (7.14)$$

$$L_n^g = \sum_{m \in \mathbb{Z}} (m - n + \hat{\mu})(m + \hat{\mu}) : \hat{B}_{m-n}^\dagger \hat{B}_m :, \quad (7.15)$$

$$\tilde{L}_n^\lambda = \sum_{r \in \mathbb{Z} + \varepsilon} (r - \frac{n}{2} - \hat{\mu}) : \tilde{\lambda}_{r-n}^\dagger \tilde{\lambda}_r :, \quad (7.16)$$

$$L_n^\lambda = \sum_{r \in \mathbb{Z} + \varepsilon} (r - \frac{n}{2} + \hat{\mu}) : \lambda_{r-n}^\dagger \lambda_r :. \quad (7.17)$$

- $\tilde{G}_r^{(+ - k)}$, $G_r^{(+ - k)}$, $\tilde{G}_r^{\lambda f}$ and $G_r^{\lambda g}$ in the super-Virasoro generators are

$$\tilde{G}_r^{(+ - k)} = \sum_{n \in \mathbb{Z}} [-\tilde{\psi}_{r-n}^+ \tilde{\alpha}_n^- - \tilde{\psi}_{r-n}^- \tilde{\alpha}_n^+ + \tilde{\psi}_{r-n}^k \tilde{\alpha}_n^k] + \frac{\mu}{\sqrt{2\alpha'}} \tilde{\psi}_r^+ (J_B + J_F), \quad (7.18)$$

$$G_r^{(+ - k)} = \sum_{n \in \mathbb{Z}} [-\psi_{r-n}^+ \alpha_n^- - \psi_{r-n}^- \alpha_n^+ + \psi_{r-n}^k \alpha_n^k] - \frac{\mu}{\sqrt{2\alpha'}} \psi_r^+ (J_B + J_F), \quad (7.19)$$

$$\tilde{G}_r^{\lambda f} = i \sum_{n \in \mathbb{Z}} (n - \hat{\mu}) (\tilde{\lambda}_{n+r} \hat{A}_n^\dagger - \tilde{\lambda}_{n-r}^\dagger \hat{A}_n), \quad (7.20)$$

$$G_r^{\lambda g} = i \sum_{n \in \mathbb{Z}} (n + \hat{\mu}) (\lambda_{n+r} \hat{B}_n^\dagger - \lambda_{n-r}^\dagger \hat{B}_n). \quad (7.21)$$

Here we define $\tilde{\alpha}_0^\pm = \alpha_0^\pm = \sqrt{\frac{\alpha'}{2}} p^\pm$, $\tilde{\alpha}_0^k = \alpha_0^k = \sqrt{\frac{\alpha'}{2}} p^k$, $\hat{A}_n = \frac{1}{\sqrt{|n-\hat{\mu}|}} A_n$, $\hat{B}_n = \frac{1}{\sqrt{|n+\hat{\mu}|}} B_n$, and moreover $\hat{\mu} = \mu \alpha' p^+$, $\varepsilon = 0$ in the R sector and $\varepsilon = \frac{1}{2}$ in the NS sector, as we define in previous section.

Calculating the commutators between the generators \tilde{L}_n^f , L_n^g , \tilde{L}_n^λ and L_n^λ , we obtain

$$[\tilde{L}_m^f, \tilde{L}_n^f] = (m - n) \tilde{L}_{m+n}^f + \tilde{A}^f(m) \delta_{m+n}, \quad (7.22)$$

$$[L_m^g, L_n^g] = (m - n) L_{m+n}^g + A^g(m) \delta_{m+n}, \quad (7.23)$$

$$[\tilde{L}_m^\lambda, \tilde{L}_n^\lambda] = (m - n) \tilde{L}_{m+n}^\lambda + \tilde{A}^\lambda(m) \delta_{m+n}, \quad (7.24)$$

$$[L_m^\lambda, L_n^\lambda] = (m - n) L_{m+n}^\lambda + A^\lambda(m) \delta_{m+n}, \quad (7.25)$$

where $\tilde{A}^f(m)$, $A^g(m)$, $\tilde{A}^\lambda(m)$ and $A^\lambda(m)$ represent the anomalies of the algebra for \tilde{L}_n^f , L_n^g , \tilde{L}_n^λ and L_n^λ ; the anomalies are

$$\tilde{A}^f(m) = A^g(m) = \frac{1}{6} (m^3 - m) - (\hat{\mu} - [\hat{\mu}]) (\hat{\mu} - [\hat{\mu}] - 1) m, \quad (7.26)$$

$$\tilde{A}^\lambda(m) = A^\lambda(m) = \begin{cases} \frac{1}{12} (m^3 - m) + \frac{1}{4} m + (\hat{\mu} - [\hat{\mu}]) (\hat{\mu} - [\hat{\mu}] - 1) m & : \text{R sector}, \\ \frac{1}{12} (m^3 - m) + (\hat{\mu} - [\hat{\mu} + \frac{1}{2}])^2 m & : \text{NS sector}, \end{cases} \quad (7.27)$$

where $[\hat{\mu}]$ is the greatest integer that is not beyond $\hat{\mu}$, namely, the Gauss' symbol. Because the twisted fields f and g are complex fields, and each of them has two degrees of freedom, the coefficient $\frac{1}{6}$ appears in Eqs.(7.26). As a known case, the anomalies of the algebra for $\tilde{L}_n^{(+ - k)}$ and $L_n^{(+ - k)}$ are $\tilde{A}^{(+ - k)}(m) = A^{(+ - k)}(m) = \frac{D-2}{8} (m^3 - 2\varepsilon m)$. Next, the anticommutators between the generators $\tilde{G}_r^{\lambda f}$ and $G_r^{\lambda g}$ are

$$\{\tilde{G}_r^{\lambda f}, \tilde{G}_s^{\lambda f}\} = 2(\tilde{L}_{r+s}^\lambda + \tilde{L}_{r+s}^f) + \tilde{B}^{\lambda f}(r) \delta_{r+s}, \quad (7.28)$$

$$\{G_r^{\lambda g}, G_s^{\lambda g}\} = 2(L_{r+s}^\lambda + L_{r+s}^g) + B^{\lambda g}(r) \delta_{r+s}, \quad (7.29)$$

where $\tilde{B}^{\lambda f}(r)$ and $B^{\lambda g}(r)$ represent the anomalies of the algebra for $\tilde{G}_r^{\lambda f}$ and $G_r^{\lambda g}$; the anomalies

are

$$\tilde{B}^{\lambda f}(r) = B^{\lambda g}(r) = \begin{cases} r^2 & : \text{R sector}, \\ r^2 - \frac{1}{4} + [\hat{\mu} + \frac{1}{2}]^2 - [\hat{\mu}]^2 - [\hat{\mu}] - 2([\hat{\mu} + \frac{1}{2}] - [\hat{\mu}] - \frac{1}{2})\hat{\mu} & : \text{NS sector}. \end{cases} \quad (7.30)$$

When we gather the super-Virasoro generators for the matter, then the super-Virasoro algebra becomes

$$[\tilde{L}_m^{\text{M}}, \tilde{L}_n^{\text{M}}] = (m-n)\tilde{L}_{m+n}^{\text{M}} + \tilde{A}^{\text{M}}(m)\delta_{m+n}, \quad [L_m^{\text{M}}, L_n^{\text{M}}] = (m-n)L_{m+n}^{\text{M}} + A^{\text{M}}(m)\delta_{m+n}, \quad (7.31)$$

$$[\tilde{L}_m^{\text{M}}, \tilde{G}_r^{\text{M}}] = \left(\frac{m}{2} - r\right)\tilde{G}_{m+r}^{\text{M}}, \quad [L_m^{\text{M}}, G_r^{\text{M}}] = \left(\frac{m}{2} - r\right)G_{m+r}^{\text{M}}, \quad (7.32)$$

$$\{\tilde{G}_r^{\text{M}}, \tilde{G}_s^{\text{M}}\} = 2\tilde{L}_{r+s}^{\text{M}} + \tilde{B}^{\text{M}}(r)\delta_{r+s}, \quad \{G_r^{\text{M}}, G_s^{\text{M}}\} = 2L_{r+s}^{\text{M}} + B^{\text{M}}(r)\delta_{r+s}, \quad (7.33)$$

where the anomalies are

$$\tilde{A}^{\text{M}}(m) = A^{\text{M}}(m) = \begin{cases} \frac{D}{8}m^3 & : \text{R sector}, \\ \frac{D}{8}(m^3 - m) + \left([\hat{\mu} + \frac{1}{2}]^2 - [\hat{\mu}]^2 - [\hat{\mu}] - 2([\hat{\mu} + \frac{1}{2}] - [\hat{\mu}] - \frac{1}{2})\hat{\mu}\right)m & : \text{NS sector}, \end{cases} \quad (7.34)$$

$$\tilde{B}^{\text{M}}(r) = B^{\text{M}}(r) = \begin{cases} \frac{D}{2}r^2 & : \text{R sector}, \\ \frac{D}{2}\left(r^2 - \frac{1}{4}\right) + [\hat{\mu} + \frac{1}{2}]^2 - [\hat{\mu}]^2 - [\hat{\mu}] - 2([\hat{\mu} + \frac{1}{2}] - [\hat{\mu}] - \frac{1}{2})\hat{\mu} & : \text{NS sector}. \end{cases} \quad (7.35)$$

Finally, we simply comment on the super-Virasoro algebra of ghosts and antighosts.^{(21), (22)} The super-Virasoro generators of them are

$$\tilde{L}_n^{\text{gh}} = \sum_{m \in \mathbb{Z}} (n+m) : \tilde{b}_{n-m} \tilde{c}_m : + \frac{1}{2} \sum_{r \in \mathbb{Z} + \varepsilon} (3n-2r) : \tilde{\gamma}_{n-r} \tilde{\beta}_r :, \quad (7.36)$$

$$L_n^{\text{gh}} = \sum_{m \in \mathbb{Z}} (n+m) : b_{n-m} c_m : + \frac{1}{2} \sum_{r \in \mathbb{Z} + \varepsilon} (3n-2r) : \gamma_{n-r} \beta_r :, \quad (7.37)$$

$$\tilde{G}_r^{\text{gh}} = -2 \sum_{s \in \mathbb{Z} + \varepsilon} \tilde{b}_{r-s} \tilde{\gamma}_s + \frac{1}{2} \sum_{s \in \mathbb{Z} + \varepsilon} (s-3r) \tilde{c}_{r-s} \tilde{\beta}_s, \quad (7.38)$$

$$G_r^{\text{gh}} = -2 \sum_{s \in \mathbb{Z} + \varepsilon} b_{r-s} \gamma_s + \frac{1}{2} \sum_{s \in \mathbb{Z} + \varepsilon} (s-3r) c_{r-s} \beta_s, \quad (7.39)$$

where \tilde{c}_n (c_n) are the oscillator modes of the left (right)-moving ghost of c^α , and \tilde{b}_n (b_n) are the oscillator modes of the left (right)-moving antighost of $b_{\alpha\beta}$, which has only two components because it is a traceless symmetric tensor, namely $b_\alpha^\alpha = 0$.^{(23), (24)} Moreover, $\tilde{\gamma}_r$ (γ_r) are the oscillator modes of the left (right)-moving ghost of γ which has spinor components, and $\tilde{\beta}_r$ (β_r) are the oscillator modes of the left (right)-moving antighost of β_α , which has only two components because it is

traceless, namely $\gamma^\alpha \beta_\alpha = 0$. (This γ^α is the two-dimensional Dirac matrix.) In terms of the modes the anticommutation relations are $\{\tilde{c}_m, \tilde{b}_n\} = \{c_m, b_n\} = \delta_{m+n}$, and the other anticommutators between $\tilde{c}_n, \tilde{b}_n, c_n$ and b_n vanish. The commutation relations are $[\tilde{\gamma}_r, \tilde{\beta}_s] = [\gamma_r, \beta_s] = \delta_{r+s}$, and the other commutators between $\tilde{\gamma}_r, \tilde{\beta}_r, \gamma_r$ and β_s vanish. Of course, the modes of c^α and $b_{\alpha\beta}$ are commutative for the modes of γ and β_α . It should be note that the anomalies $\tilde{A}^{\text{gh}}(m)$ and $A^{\text{gh}}(m)$ of the algebras $[\tilde{L}_m^{\text{gh}}, \tilde{L}_n^{\text{gh}}]$ and $[L_m^{\text{gh}}, L_n^{\text{gh}}]$ are $-\frac{5}{4}m^3$ in the R sector and $-\frac{5}{4}m^3 + \frac{1}{4}m$ in the NS sector. In addition, the anomalies of $\tilde{B}^{\text{gh}}(r)$ and $B^{\text{gh}}(r)$ of the algebras $\{\tilde{G}_r, \tilde{G}_s\}$ and $\{G_r, G_s\}$ are $-5r^2$ in the R sector and $-5r^2 + \frac{1}{4}$ in the NS sector.

§8. The nilpotency of the BRST charge

In the type II superstring theory, the left modes and the right modes are independent. Therefore, the BRST charge can be decomposed into the left modes and the right modes as

$$Q_B = Q_B^L + Q_B^R, \quad (8.1)$$

where

$$Q_B^L = \sum_{m \in \mathbb{Z}} : \left[\tilde{L}_m^M + \frac{1}{2} \tilde{L}_m^{\text{gh}} - a \delta_{m,0} \right] \tilde{c}_{-m} : + \sum_{r \in \mathbb{Z} + \varepsilon} : \left[\tilde{G}_r^M + \frac{1}{2} \tilde{G}_r^{\text{gh}} \right] \tilde{\gamma}_{-r} : , \quad (8.2)$$

$$Q_B^R = \sum_{m \in \mathbb{Z}} : \left[L_m^M + \frac{1}{2} L_m^{\text{gh}} - a \delta_{m,0} \right] c_{-m} : + \sum_{r \in \mathbb{Z} + \varepsilon} : \left[G_r^M + \frac{1}{2} G_r^{\text{gh}} \right] \gamma_{-r} : . \quad (8.3)$$

Here a is an ordering constant. Concentrating our attention on the normal ordering of the mode operators, especially with regard to the ghosts, we obtain the square of Q_B :

$$\begin{aligned} Q_B^2 &= \frac{1}{2} \left[\{Q_B^L, Q_B^L\} + \{Q_B^R, Q_B^R\} \right] \\ &= \frac{1}{2} \sum_{m,n \in \mathbb{Z}} \left[\left([\tilde{L}_m, \tilde{L}_n] - (m-n) \tilde{L}_{n+m} \right) \tilde{c}_{-m} \tilde{c}_{-n} + \left([L_m, L_n] - (m-n) L_{n+m} \right) c_{-m} c_{-n} \right] \\ &\quad + \frac{1}{2} \sum_{r,s \in \mathbb{Z} + \varepsilon} \left[\left(\{\tilde{G}_r, \tilde{G}_s\} - 2 \tilde{L}_{r+s} \right) \tilde{\gamma}_{-r} \tilde{\gamma}_{-s} + \left(\{G_r, G_s\} - 2 L_{r+s} \right) \gamma_{-r} \gamma_{-s} \right] \\ &= \frac{1}{2} \sum_{m \in \mathbb{Z}} A(m) (\tilde{c}_{-m} \tilde{c}_m + c_{-m} c_m) + \frac{1}{2} \sum_{m \in \mathbb{Z} + \varepsilon} B(m) (\tilde{\gamma}_{-m} \tilde{\gamma}_m + \gamma_{-m} \gamma_m) , \end{aligned} \quad (8.4)$$

where $\tilde{L}_m, L_m, \tilde{G}_r$ and G_r are the total super-Virasoro generators as follows:

$$\tilde{L}_m = \tilde{L}_m^M + \tilde{L}_m^{\text{gh}} - a \delta_{m,0} , \quad L_m = L_m^M + L_m^{\text{gh}} - a \delta_{m,0} , \quad (8.5)$$

$$\tilde{G}_r = \tilde{G}_r^M + \tilde{G}_r^{\text{gh}} , \quad G_r = G_r^M + G_r^{\text{gh}} . \quad (8.6)$$

$A(m)$ and $B(r)$ are the total anomalies from the total super-Virasoro generators, and we can exactly demonstrate them using the results of the anomalies of the matter in the previous section and the

known anomalies of the ghosts. As a result, the total anomalies are

$$A(m) = \begin{cases} \frac{D-10}{8}m^3 + 2am & : \text{R sector}, \\ \frac{D-10}{8}(m^3 - m) + 2\left(a - \frac{1}{2} + [\hat{\mu} + \frac{1}{2}]^2 - [\hat{\mu}]^2 - [\hat{\mu}] - 2([\hat{\mu} + \frac{1}{2}] - [\hat{\mu}] - \frac{1}{2})\hat{\mu}\right)m & : \text{NS sector}, \end{cases} \quad (8.7)$$

$$B(r) = \begin{cases} \frac{D-10}{2}r^2 + 2a & : \text{R sector}, \\ \frac{D-10}{2}(r^2 - \frac{1}{4}) + 2\left(a - \frac{1}{2} + [\hat{\mu} + \frac{1}{2}]^2 - [\hat{\mu}]^2 - [\hat{\mu}] - 2([\hat{\mu} + \frac{1}{2}] - [\hat{\mu}] - \frac{1}{2})\hat{\mu}\right) & : \text{NS sector}. \end{cases} \quad (8.8)$$

If the anomalies are zero, the square of the BRST charge vanishes. Because the BRST charge must have the property of nilpotency, the anomalies must be zero. Thus we can determine the number of spacetime dimensions and the ordering constant in the pp -wave background with the NS-NS flux:

$$D = 10, \quad a = 0 \quad : \text{R sector}, \quad (8.9)$$

$$D = 10, \quad a = \frac{1}{2} - [\hat{\mu} + \frac{1}{2}]^2 + [\hat{\mu}]^2 + [\hat{\mu}] + 2([\hat{\mu} + \frac{1}{2}] - [\hat{\mu}] - \frac{1}{2})\hat{\mu} \quad : \text{NS sector}. \quad (8.10)$$

Considering the spectrum of the superstring in the pp -wave background, the physical state must satisfy $Q_B|\text{phys}\rangle = 0$. Using our free-modes, we can get them.

§9. Conclusion

In this paper we have canonically quantized the closed RNS superstring in the pp -wave background with the non-zero flux of the $B_{\mu\nu}$ field using the covariant BRST operator formalism. In this pp -wave background with the flux, we have constructed the general operator solutions and the free-mode representations of all the covariant string coordinates and fermions. Moreover, we proved that the free-mode representations satisfy both the equal-time canonical (anti)commutation relations between all the covariant superstring fields and the Heisenberg equations of motion, whose form is the same as that of the Euler-Lagrange equations of motion in the pp -wave background with the flux. It is worth noting that the zero mode x^- of X_0^- has played important roles in this study. Since the energy-momentum tensor and the supercurrent take very simple forms in the free-mode representations of the covariant string coordinates and fermions, we have been able to calculate the anomaly in the super-Virasoro algebra. Using this anomaly, we have determined the number of dimensions of spacetime and the ordering constant from the nilpotency condition of the BRST charge in the pp -wave background with the flux. The spacetime supersymmetry is realized due to the condition like the GSO projection based on the difference between the R sector and the NS sector.

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